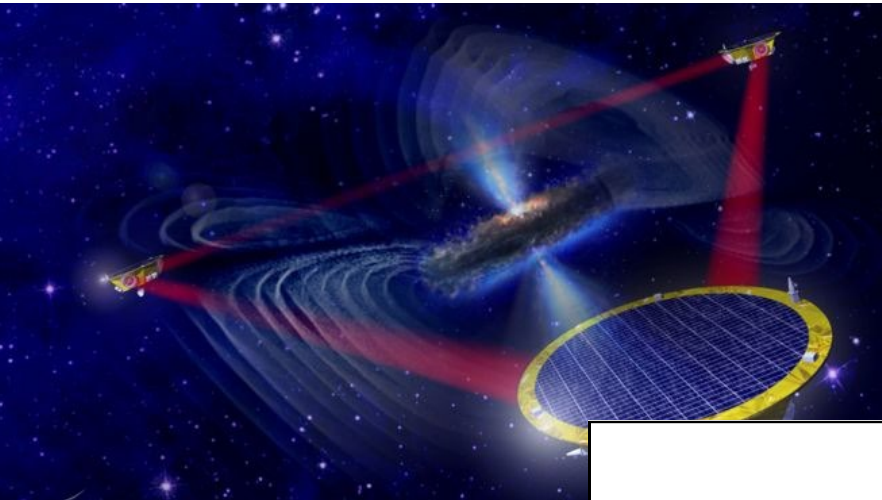
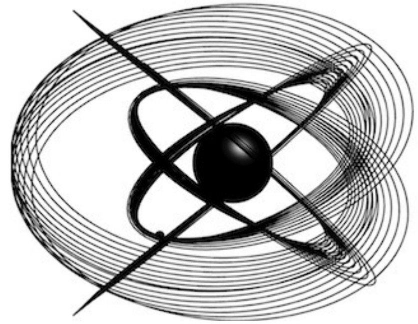
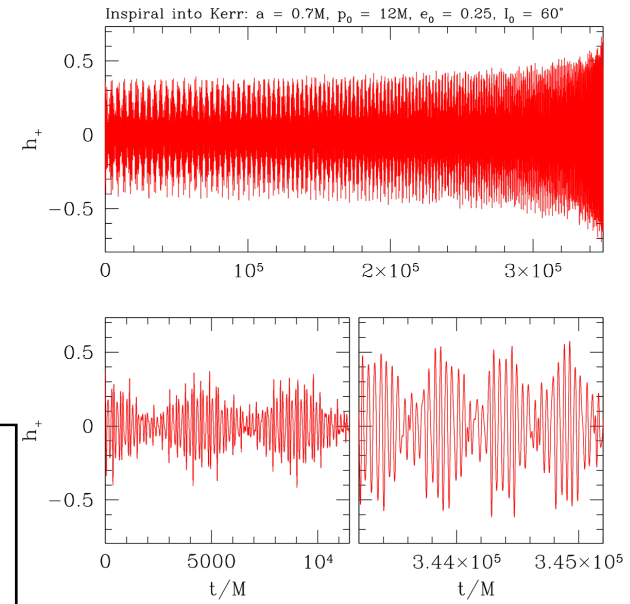
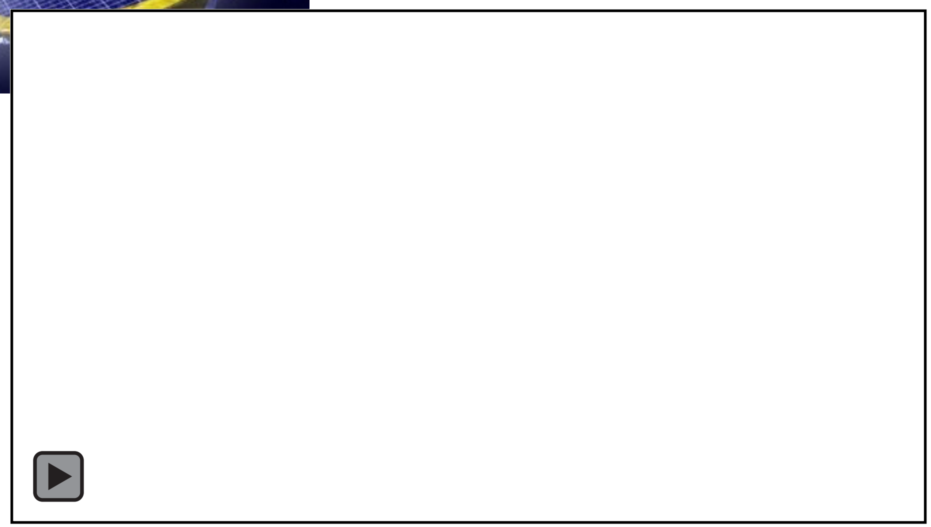
Small mass-ratio binary modeling









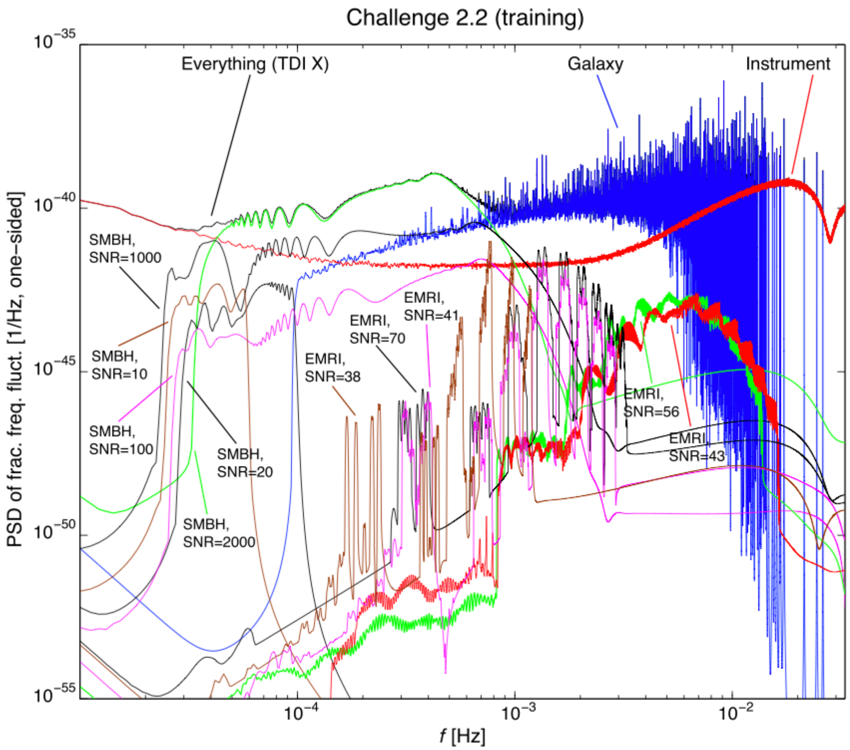
Making EMRI waveforms for LISA great again

(Blame Lorenzo for the subtitle!!)

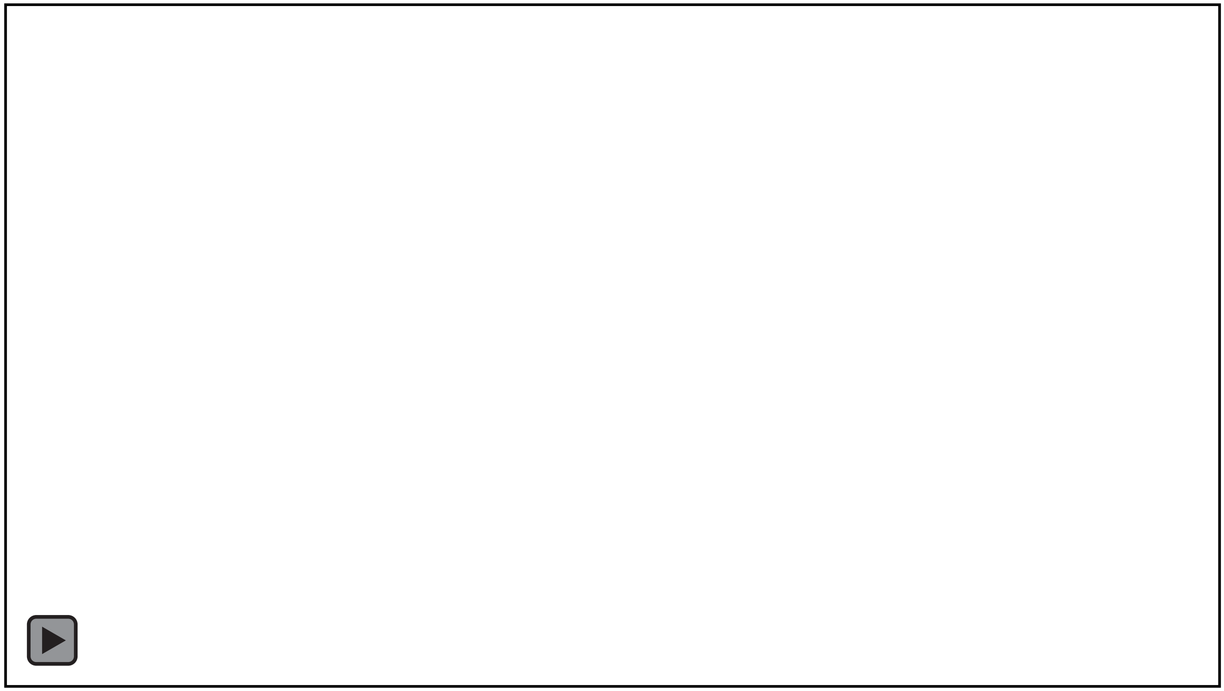
Scott A. Hughes, MIT & Lorenzo Speri, ESA

10 Years to LISA, 1-3 April 2025

Canonical EMRI characteristics



Old MLDC graphic: Arnaud et al, CQG **24** S551 (2007).



Video credit: Steve Drasco

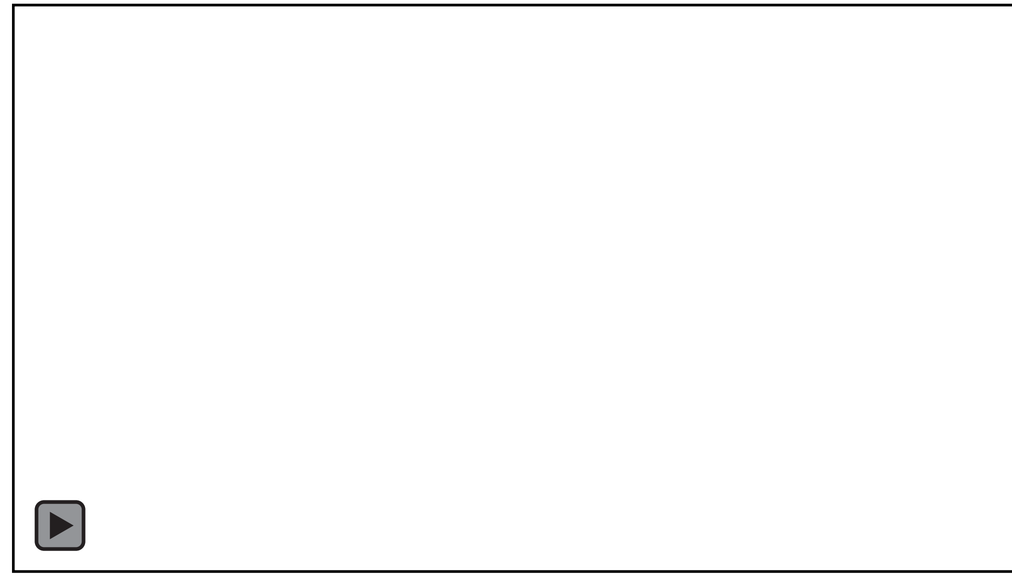
EMRI waveform is long and complicated: encodes lots of information about the system, but waveform is below noise, and weaker than most LISA signals.

Need good waveform models!

Kludges

Long ago recognized need for waveforms which capture time-frequency structure of EMRI orbits.

Key: Triperiodic structure, different frequencies associated with motions in radius (Ω_r), polar angle (Ω_θ), and azimuth (Ω_φ).



Couple this with an approximation to make waveforms, can compute models quite similar to relativistic EMRI models … but much more quickly.

Kludges

Long ago recognized need for waveforms which capture time-frequency structure of EMRI orbits.

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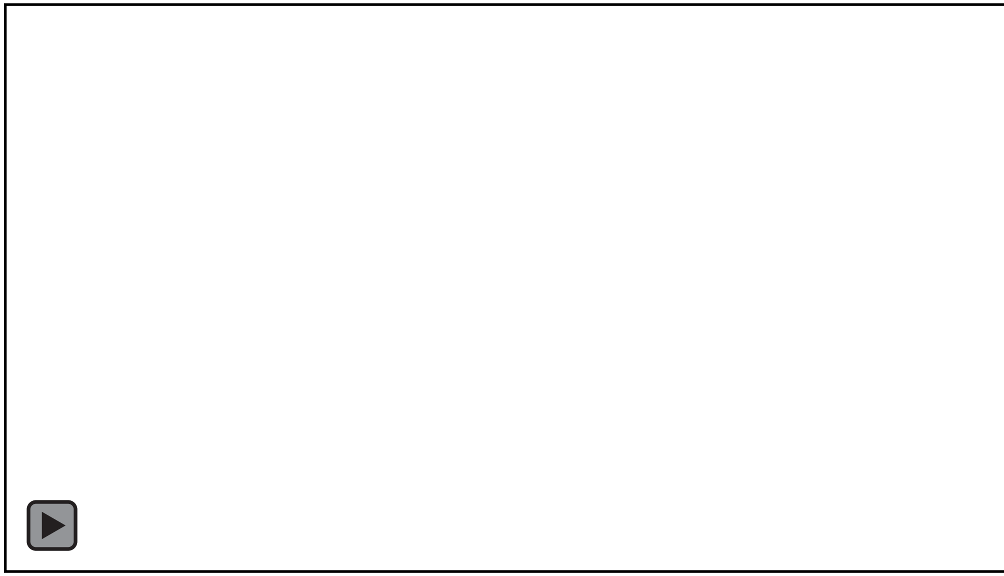

Excellent tools for exploring EMRI measurement and data analysis … but miss a lot of physics, and not useful for accurate data fit.

Relativistic EMRI waveforms

To make the waveforms "correctly," we need to solve Einstein field equations … small mass ratio gives us tools for doing this relatively easily.

$$g_{\alpha\beta} = g^{\mathrm{Kerr}}_{\alpha\beta} + \epsilon h^{(1)}_{\alpha\beta} + \epsilon^2 h^{(2)}_{\alpha\beta} + \ldots \qquad \epsilon \equiv \frac{m}{M}$$

Exact solution

Perturbations

Conceptually useful to regard motion as orbit of Kerr spacetime, corrected by these forcing terms:

$$\frac{dJ_i}{d\mathrm{T}} = 0 + \epsilon G_i^{(1)}(q_r, q_\theta, J_k) + \epsilon^2 G_i^{(2)}(q_r, q_\theta, J_k) + \cdots$$

"dissipative"

J_i = (E, L, Q) = constants of motion that characterize black hole orbits

Relativistic EMRI waveforms

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$$\frac{dq_\alpha}{d\mathrm{T}} = \Omega_\alpha(J_k) + \epsilon g_\alpha^{(1)}(q_r, q_\theta, J_k) + \epsilon^2 g_\alpha^{(2)}(q_r, q_\theta, J_k) + \cdots \quad \text{"conservative"}$$

$q_a = (q_t, q_r, q_\theta, q_\varphi)$ = "angles" describing each coordinate degree of freedom.

Two-timescale expansion

System characterized by "short" timescale T_{orb} describing orbits and a "long" timescale T_{GW} describing how orbits change due to GWs.

From self forces, not hard to show that $T_{\mathrm{orb}}/T_{\mathrm{GW}} \sim m/M$ …
can then see how phase of EMRI waveforms accumulates:

$$\Phi(t_1, t_2) = \int_{t_1}^{t_2} \omega(t)\, dt = \Phi_{\mathrm{diss}-1} + \Phi_{\mathrm{cons}-1} + \Phi_{\mathrm{diss}-2}$$

"Adiabatic" term: Frequencies ($\sim 1/T_{\mathrm{orb}}$) integrated over inspiral time T_{GW}.

Scales inversely with mass ratio.

Two-timescale expansion

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"Post-adiabatic" terms: Conservative correction to frequencies integrated over inspiral time, plus 2nd order dissipative correction to inspiral time integrated over frequencies.

Result is independent of mass ratio.

Two-timescale expansion

System characterized by "short" timescale $T_{\rm orb}$ describing orbits and a "long" timescale $T_{\rm GW}$ describing how orbits change due to GWs.

From self forces, not hard to show that $T_{\rm orb}/T_{\rm GW} \sim m/M$ … can then see how phase of EMRI waveforms accumulates:

$$\Phi(t_1, t_2) = \int_{t_1}^{t_2} \omega(t)\, dt = \Phi_{\rm diss-1} + \Phi_{\rm cons-1} + \Phi_{\rm diss-2}$$

Completely understand how to compute leading dissipative term: Work needed to implement it, but adiabatic waveform development is well underway.

Adiabatic waveforms

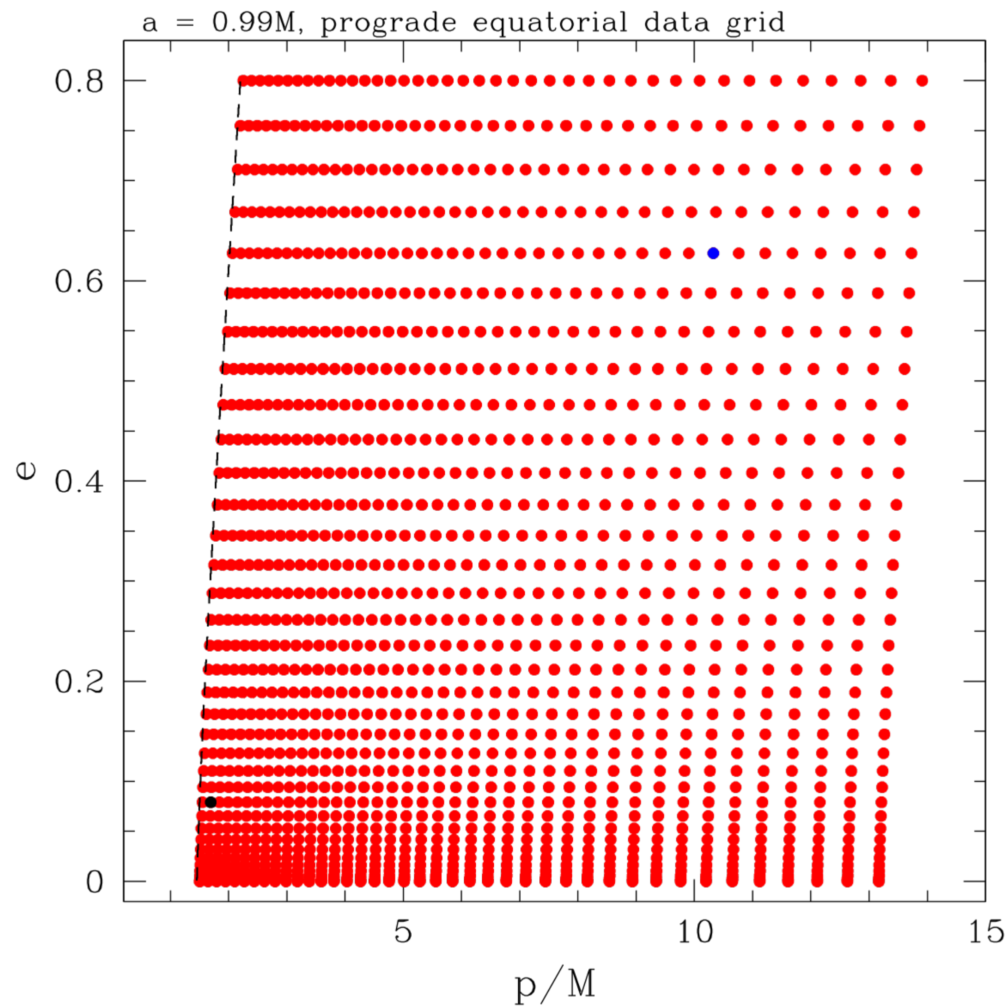
Big effort: compute data for self interaction on dense grid of orbits covering astrophysical space.

Preliminary example: Grid in (p, e) plane covering strong-field region near black hole of spin a = 0.99M.

Parameters p and e describe eccentric equatorial orbits:

$$r = \frac{p}{1 + e\cos(\chi_r)}$$

Anomaly angle describes where smaller body is in its orbit (0 = periapsis, π = apoapsis).

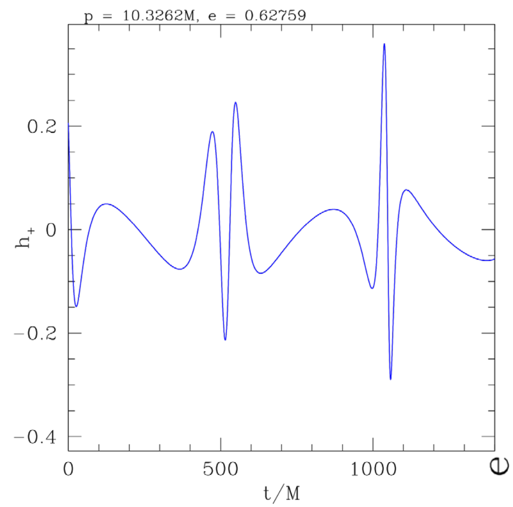


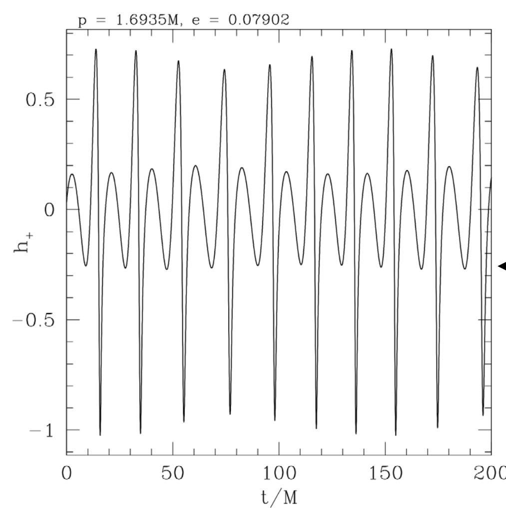
(12,914,342 harmonics over 1440 orbits)

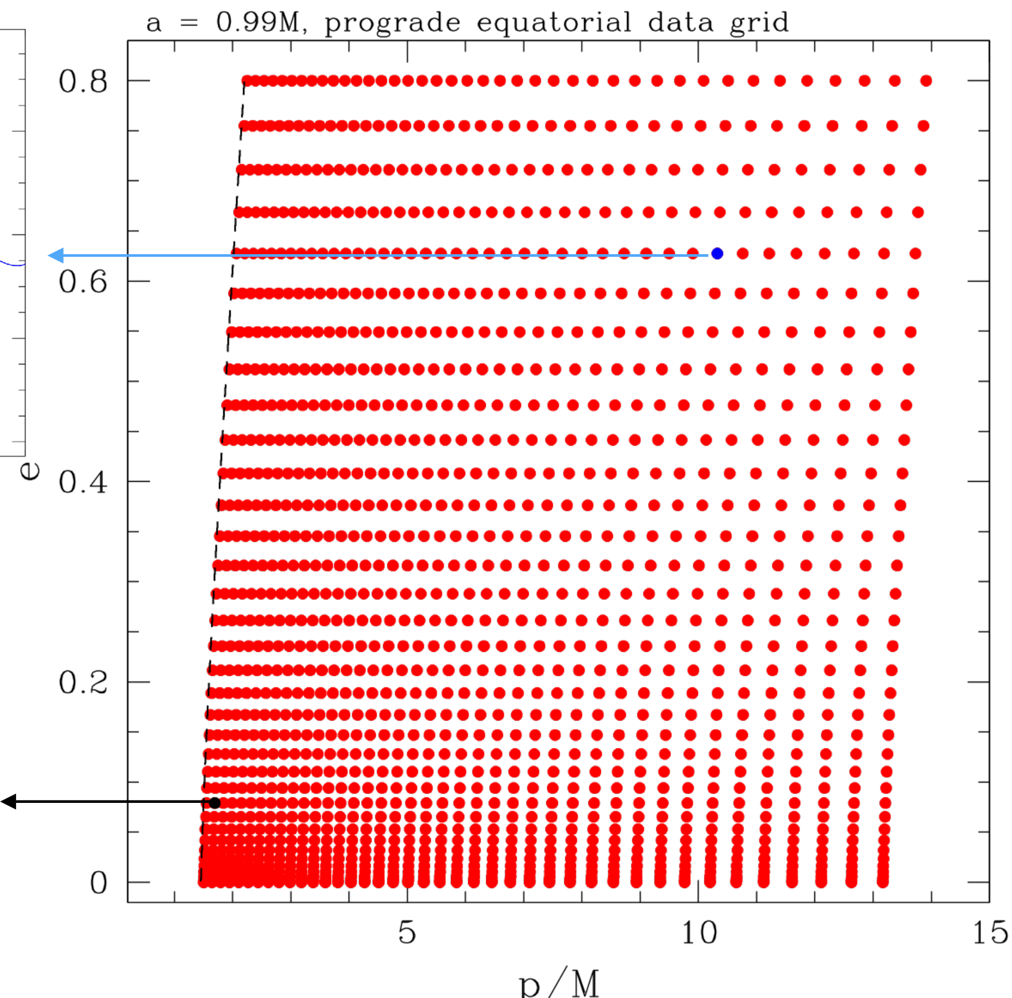
Adiabatic waveforms

Big effort: compute data for self interaction on dense grid of orbits covering astrophysical space.

Waveform snapshots at points in data grid illustrate how amplitude and frequency change as we move around in eccentricity and mean separation.







(12,914,342 harmonics over 1440 orbits)

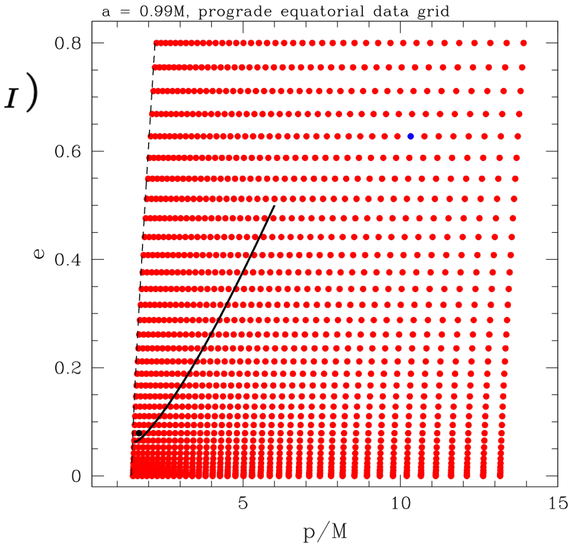
Adiabatic waveforms

Start at some initial point; use data to evolve to the last stable orbit; "stitch" together waveform on that inspiral that results from this evolution.

$$h_+(t) - ih_\times(t) = \frac{1}{r}\sum_{\ell mkn} H_{\ell mkn}(p,e,x_I)Y_{\ell m}(\theta;\phi)e^{-i\Phi_{mkn}(p,e,x_I)}$$

$$(p,e,x_I) = [p(t), e(t), x_I(t)]$$

$$\Phi_{mkn}(p,e,x_I) = \int_{t_0}^{t}\left[m\Omega_\phi(t') + k\Omega_\theta(t') + n\Omega_r(t')\right]dt'$$

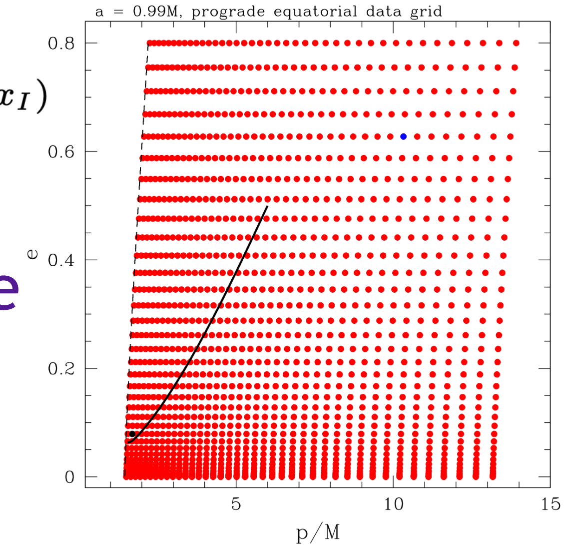


With the data in hand, straightforward to assemble waveform in both time and frequency domain.

Adiabatic waveforms

Start at some initial point; use data to evolve to the last stable orbit; "stitch" together waveform on that inspiral that results from this evolution.

$$\tilde{h}_+(t) - i\tilde{h}_\times(f) = \frac{1}{r}\sum_{\ell mkn} H_{\ell mkn}(p,e,x_I)Y_{\ell m}(\theta;\phi)\mathcal{F}_{\ell mkn}(f)e^{-i\Phi_{mkn}(p,e,x_I)}$$

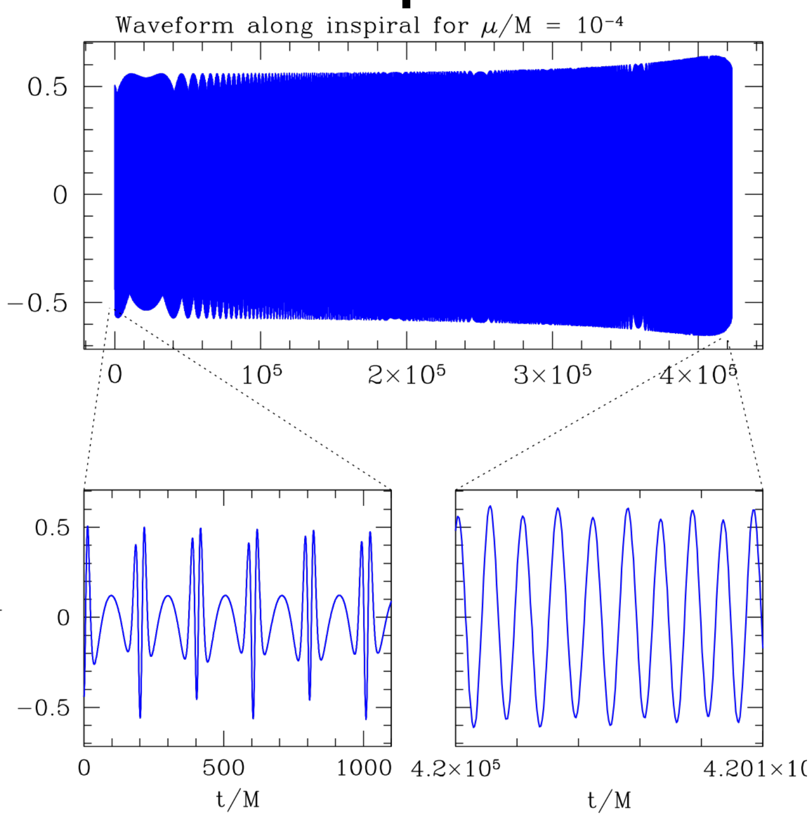


Use waveform's predictable time-frequency structure to map between time and frequency domains. (Weight function F_{lmkn} accounts for fact that inspiral spends more time at low frequency than at high.)

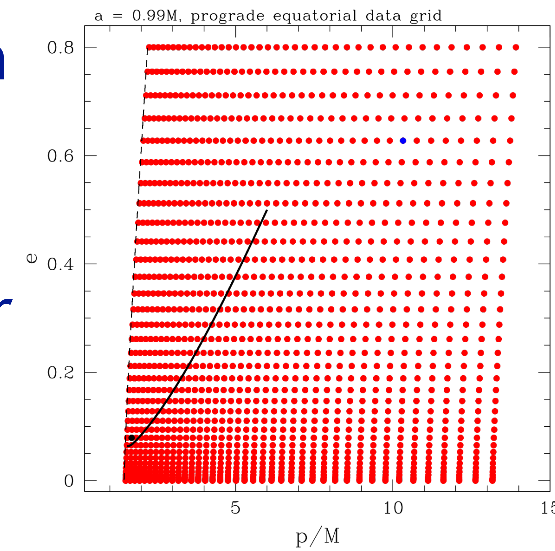
With the data in hand, straightforward to assemble waveform in both time and frequency domain.

Adiabatic waveforms

Start at some initial point; use data to evolve to the last stable orbit; "stitch" together waveform on that inspiral that results from this evolution.



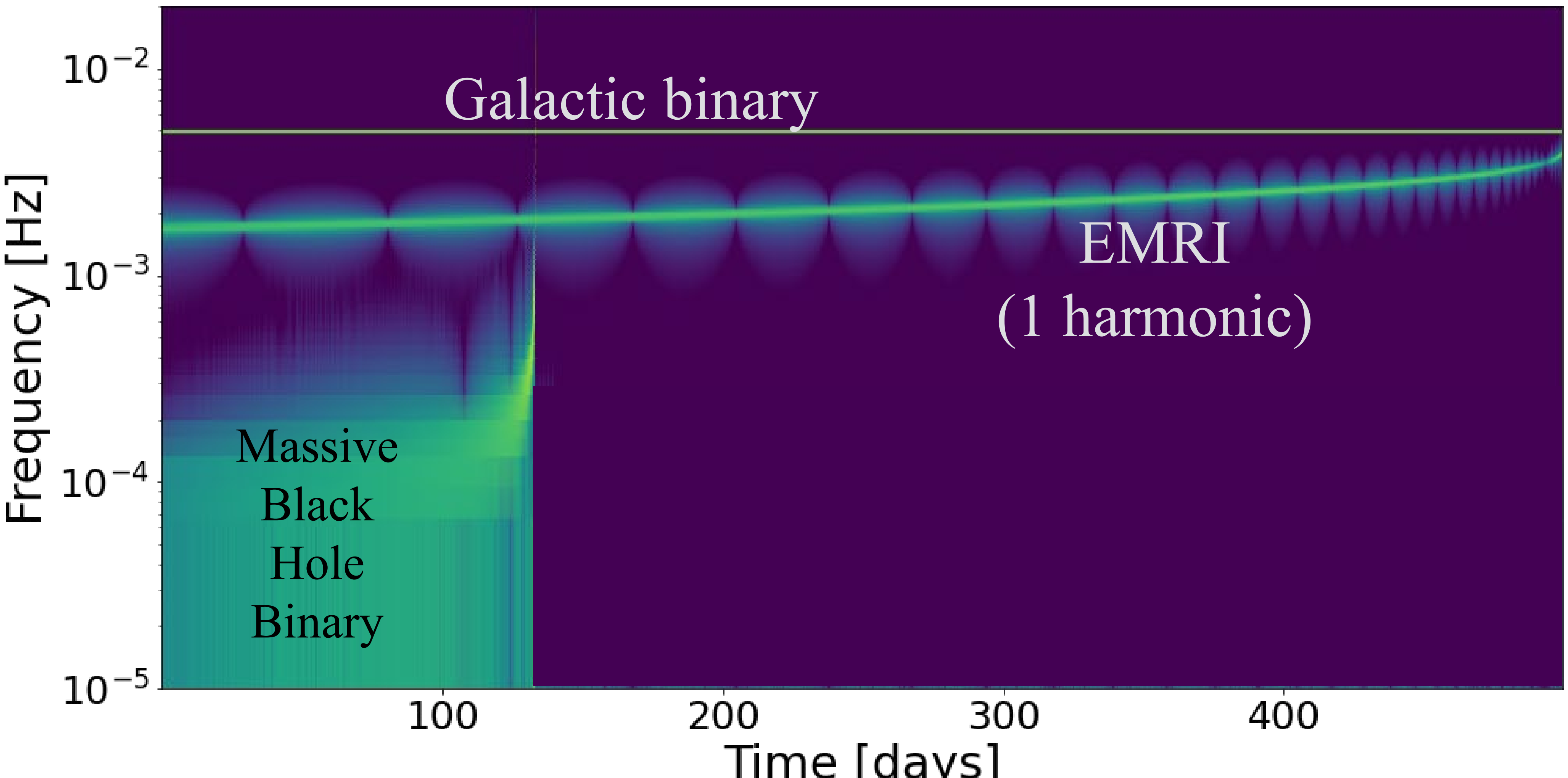
This example: starts with substantial eccentricity, visible in waveform's early cycles. Much closer to circular as system reaches last stable orbit.



Preliminary datasets for equatorial eccentric and circular inclined inspiral (covering spins *a*/*M* = 0, 0.1, 0.2, …, 0.8, 0.9, 0.95, 0.99) have been publicly released … currently developing denser grids for "production" level equatorial eccentric waveforms, denser in all directions.

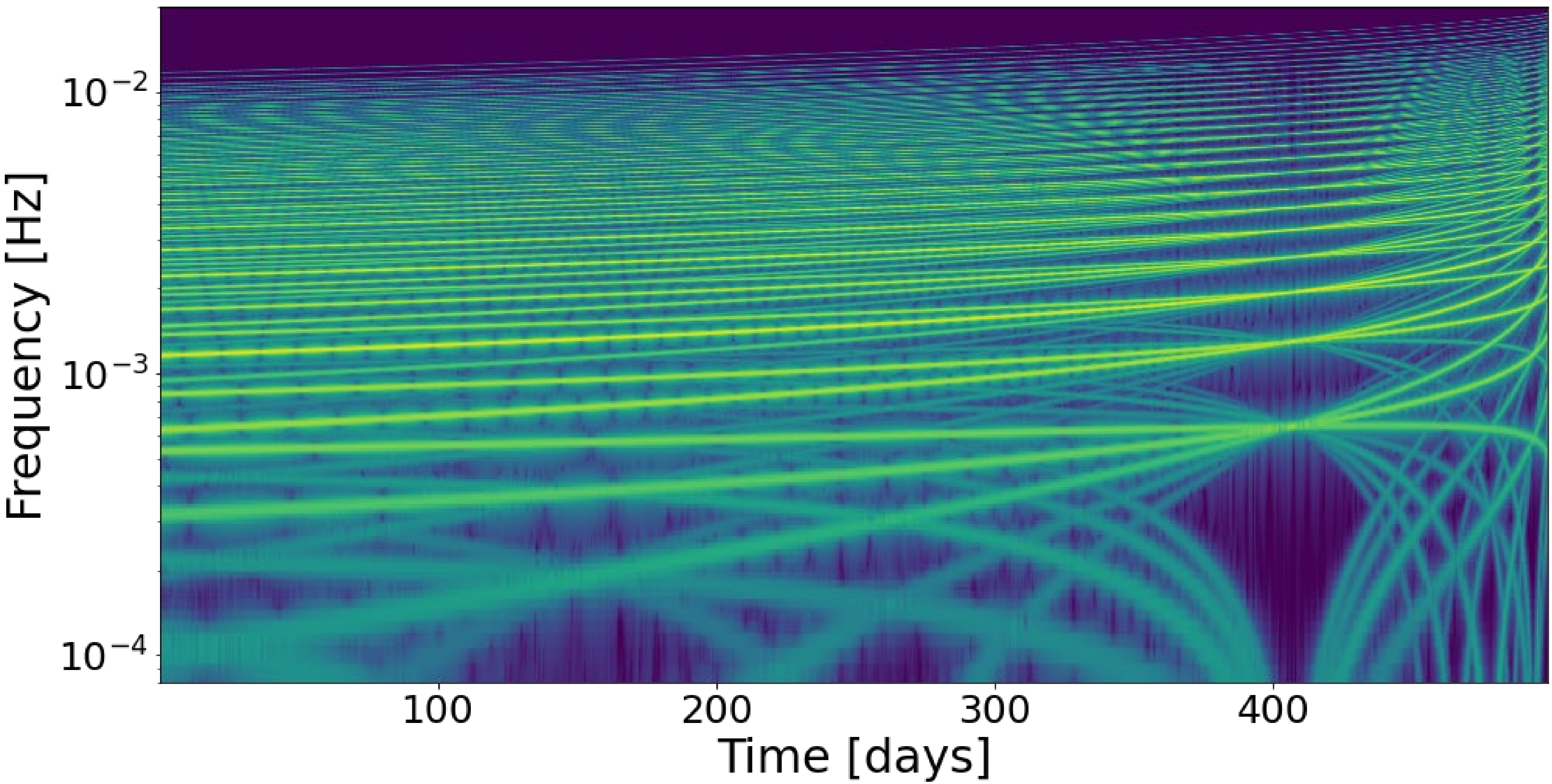
Time-frequency structure

Each multipolar "voice" which contributes to the waveform is quite simple



Time-frequency structure

The chorus of *all* voices is **much** more complicated!



Computational cost

Large up-front cost (about 10^3 CPU hours in example shown), but embarrassingly parallelizable … once data computed, can make waveforms very quickly.

That complicated structure is a sum of contributions (“voices”) that are individually quite simple.

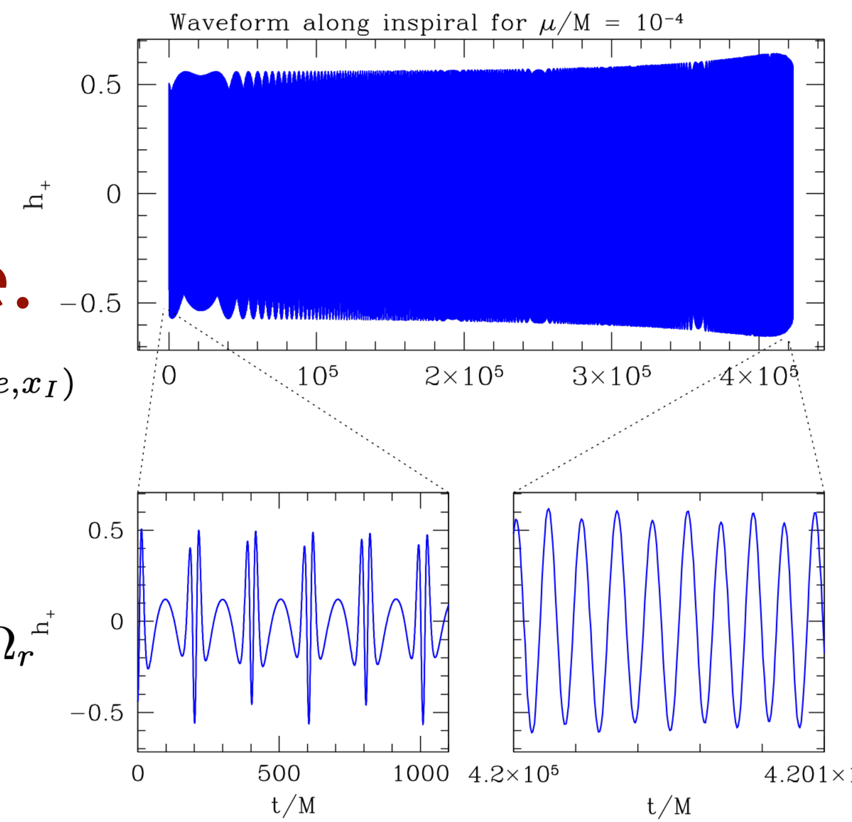
$$h_+(t) - ih_\times(t) = \frac{1}{r}\sum_{\ell mkn} H_{\ell mkn}(p,e,x_I)Y_{\ell m}(\theta;\phi)e^{-i\Phi_{mkn}(p,e,x_I)}$$

Complicated

$$\Phi_{mkn} = \int_0^t \omega_{mkn}(t')dt'$$

$$\omega_{mkn} = m\Omega_\phi + k\Omega_\theta + n\Omega_r$$

Smooth and simple

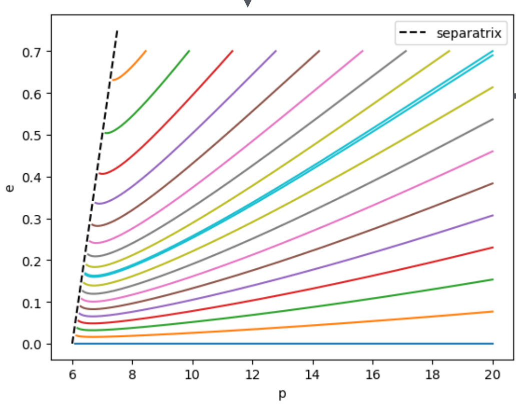


Computational cost

Exploit smoothness to quickly assemble the quantities which go into the waveform

$$h_+(t) - ih_\times(t) = \frac{1}{r}\sum_{\ell mkn} H_{\ell mkn}(p,e,x_I)Y_{\ell m}(\theta;\phi)e^{-i\Phi_{mkn}(p,e,x_I)}$$

Input Parameters
(M, μ, a, p_0, e_0, x_{I0})
(r, θ, φ, ψ_{r0}, $\psi_{\theta 0}$)



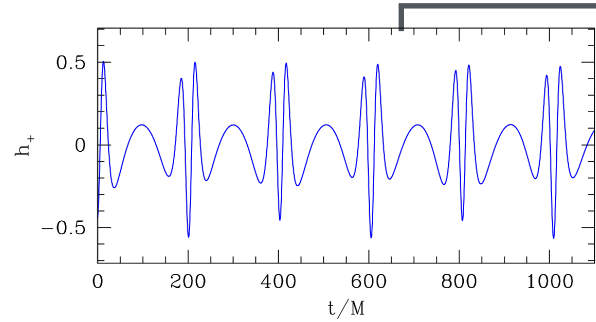
Hughes+ 2021

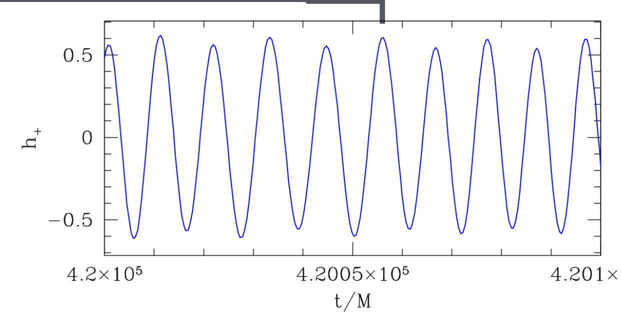
$(\Phi_r, \Phi_\theta, \Phi_\phi)$

$\{H_{\ell mkn}\}$

Domain Choice
(TD, FD, Wavelet)

Waveform Build





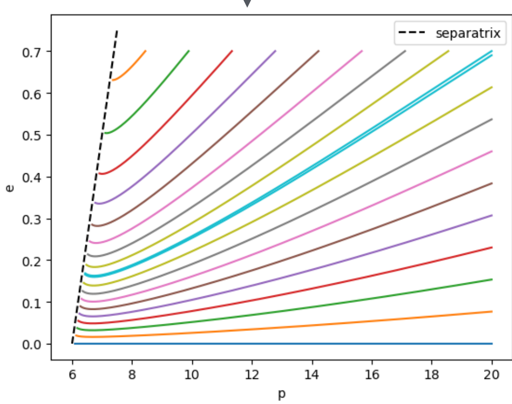
“Brute force”: Waveform generated in minutes to hours.

Computational cost

Exploit smoothness to quickly assemble the quantities which go into the waveform

$$h_+(t) - ih_\times(t) = \frac{1}{r}\sum_{\ell mkn} H_{\ell mkn}(p,e,x_I)Y_{\ell m}(\theta;\phi)e^{-i\Phi_{mkn}(p,e,x_I)}$$

Input Parameters
$(M,\ \mu,\ a,\ p_0,\ e_0,\ x_{I0})$
$(r,\ \theta,\ \varphi,\ \psi_{r0},\ \psi_{\theta0})$



$(\Phi_r, \Phi_\theta, \Phi_\phi)$

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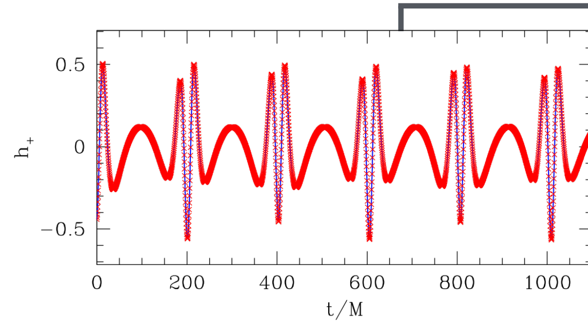

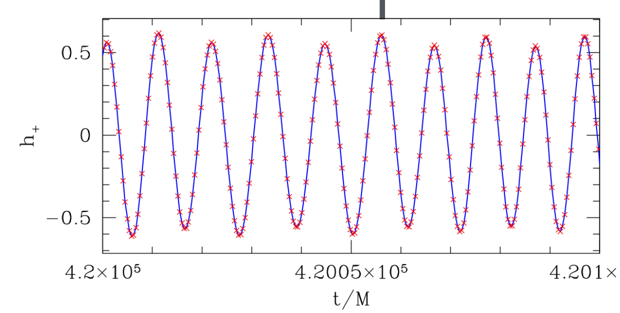

Chua+ 2021, Katz+ 2021, Speri+ 2023, Chapman-Bird+ in prep.

Take advantage of smoothness, use GPUs: 10s-100s of milliseconds.

Computational cost

Exploit smoothness to quickly assemble the quantities which go into the waveform

$$h_+(t) - i h_\times(t) = \frac{1}{r}\sum_{\ell mkn} H_{\ell mkn}(p,e,x_I)Y_{\ell m}(\theta;\phi)e^{-i\Phi_{mkn}(p,e,x_I)}$$

Input Parameters
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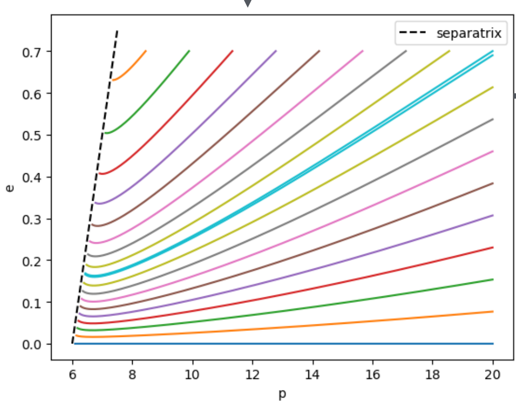

$(\Phi_r, \Phi_\theta, \Phi_\phi)$

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(TD, FD, Wavelet)

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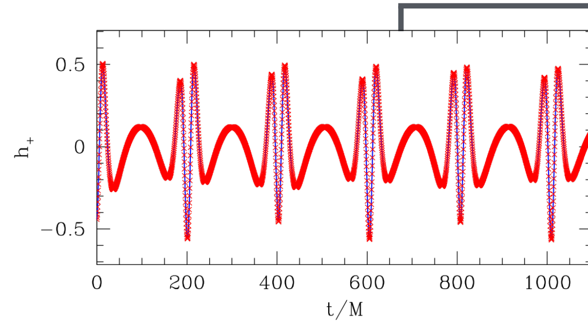
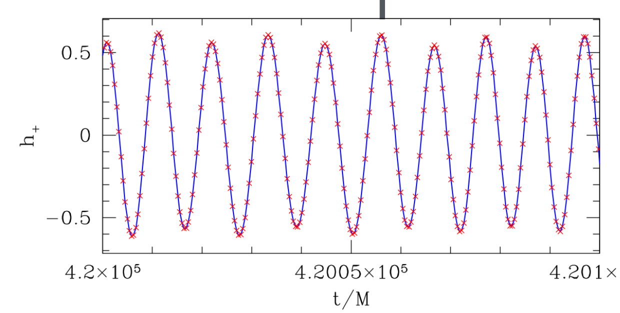




bhptoolkit.org/FastEMRIWaveforms

Take advantage of smoothness, use GPUs: 10s-100s of milliseconds.

Applications

Can do science studies with relativistic waveforms

Example (from Chapman-Bird+, in prep): inspiral of 10 $M_\odot$ into 10^6 $M_\odot$ with spin a = 0.998M, e_0 = 0.73, p_0 = 7.73M.

1-σ error bars:

- On mass M: ±2.5$M_\odot$
- On mass μ: ±3.1×$10^{-5}$$M_\odot$
- On spin a: ±7.2×10^{-7}
- On initial e: ±4.6×10^{-7}

(Note: error on μ equal to 70% of the mass of Uranus, or 10x Earth mass)

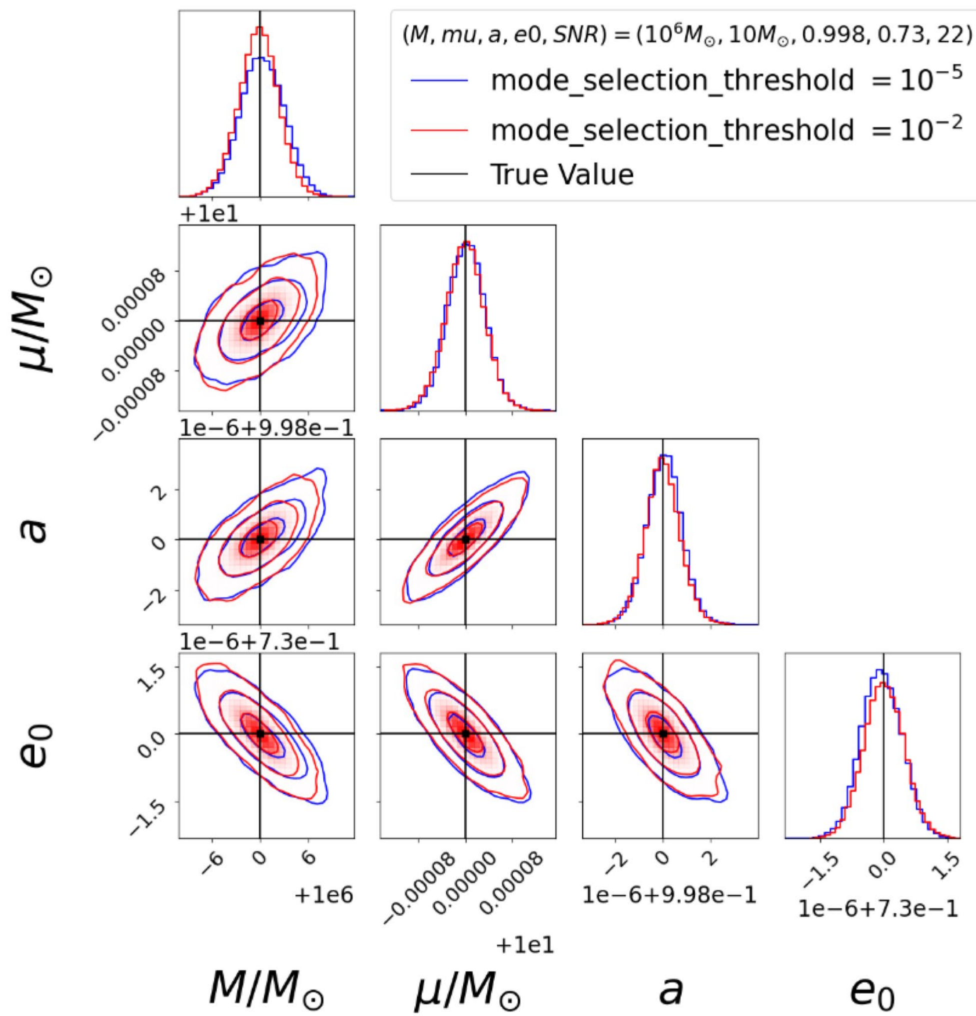


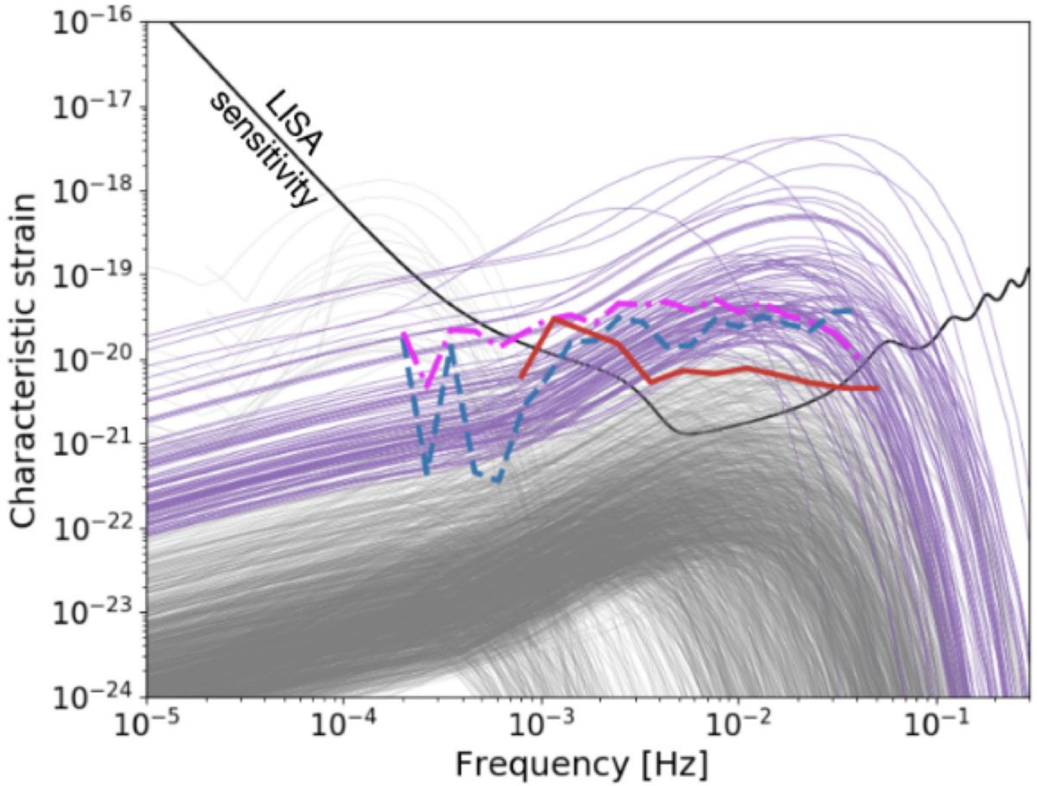
Figure and details courtesy Ollie Burke

Some issues

Current technique for amplitude data is challenged by high eccentricity: $e > 0.9$ is very difficult.

Recent analysis of Naoz and Haiman predict a "rain" of EMRIs (as well as stars) onto the central BH following a galaxy merger.

Mean eccentricity in this model: ***e*** **= 0.996.**

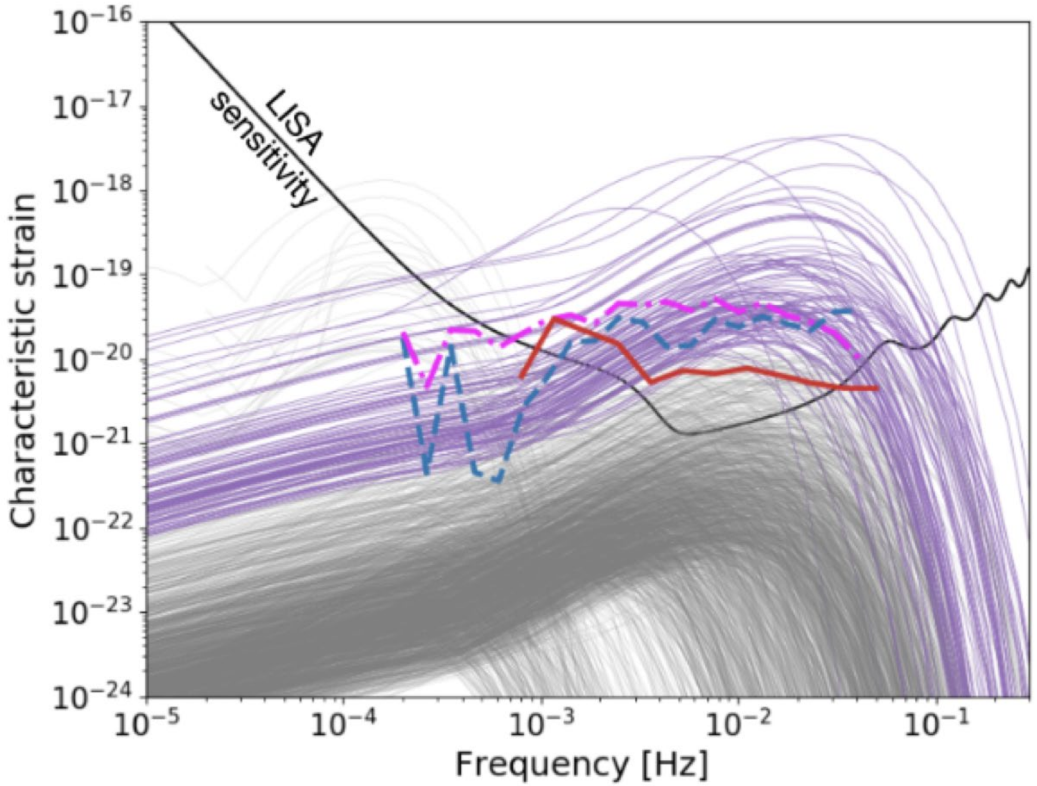


S. Naoz and Z. Haiman, ApJL **955**, L27 (2023); arXiv:2307.11149
See also B. Rom et al, ApJ **977**, 7 (2024); arXiv:2406.19443

Some issues

Current technique for amplitude data is challenged by high eccentricity: $e > 0.9$ is very difficult.

Independent of model details, this serves to remind us: We must be prepared for what we currently consider to be “edge cases.” Nature determines where the edges are, not our waveform codes!



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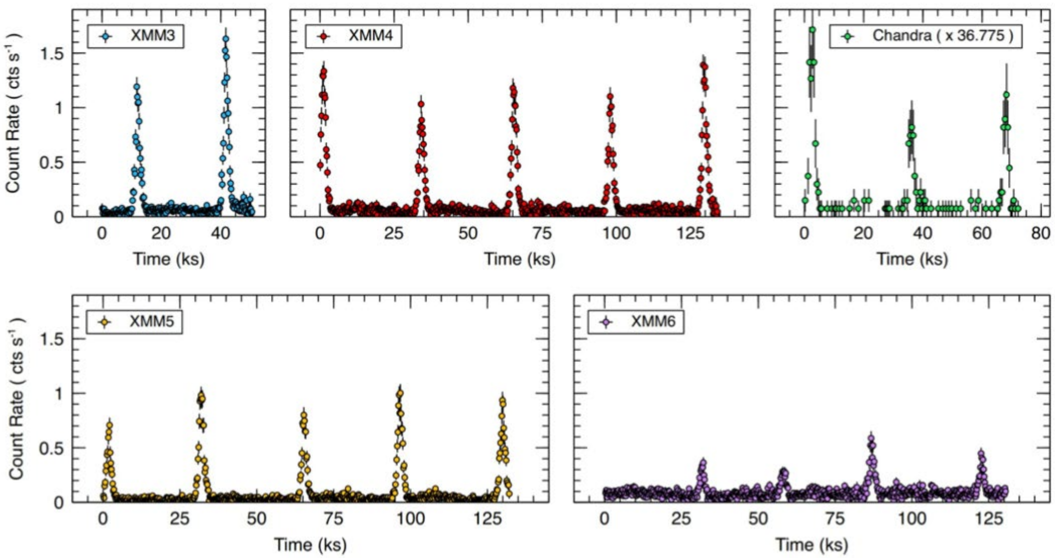
Some issues

Evidence is building that at least some quasi-periodic eruptions are associated with EMRI-like orbits interacting with accreting material.



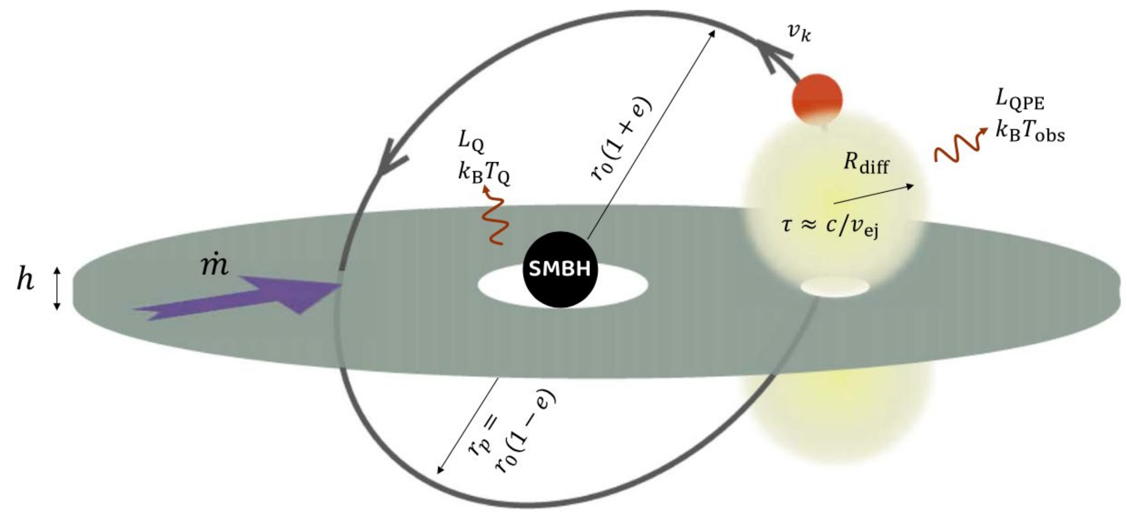
Miniutti et al., arXiv:2207.07511;
Astron. & Astrophys. **670**, A93 (2023).

Source GSN 069 started out as a tidal disruption event in 2019 … but has since flashed in x-rays every 9 hours.

Some issues

Evidence is building that at least some quasi-periodic eruptions are associated with EMRI-like orbits interacting with accreting material.

Hypothesis: orbiting compact body interacting with a disk of material! Original event disruption of a giant star; WD core now orbiting and striking the remnant disk.

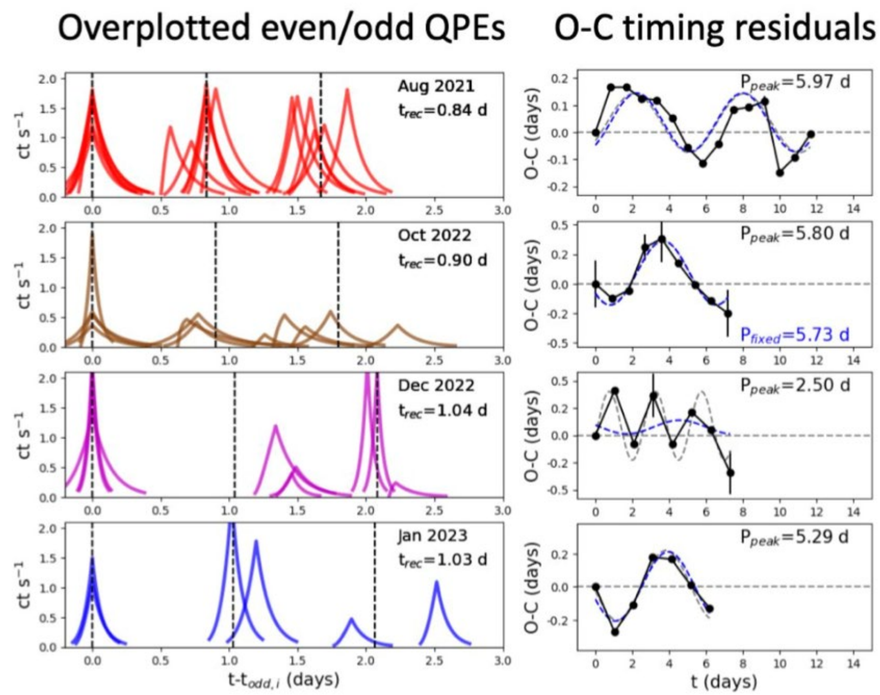


Linial and Metzger, arXiv:2303.16231; ApJ **957**, 34L (2023).

Some issues

Evidence is building that at least some quasi-periodic eruptions are associated with EMRI-like orbits interacting with accreting material.

Another example: eRO-QPE1 now monitored for 3.5 years using *NICER*. Regular eruption events show periodic residuals from a simple orbit model.

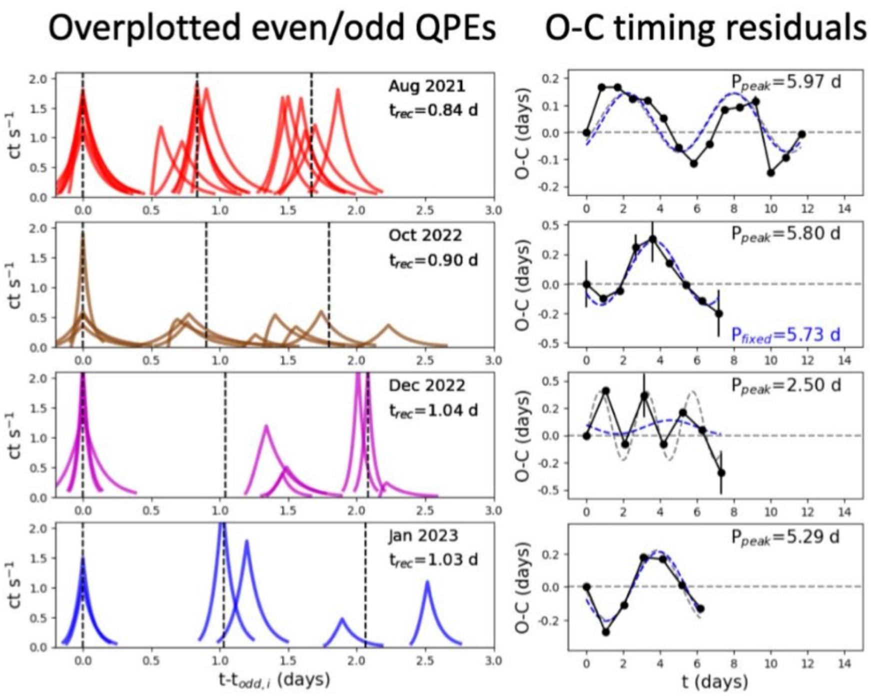


J. Chakraborty et al., arXiv:2402.08722; ApJ **965**, 12C (2024).

Some issues

Evidence is building that at least some quasi-periodic eruptions are associated with EMRI-like orbits interacting with accreting material.

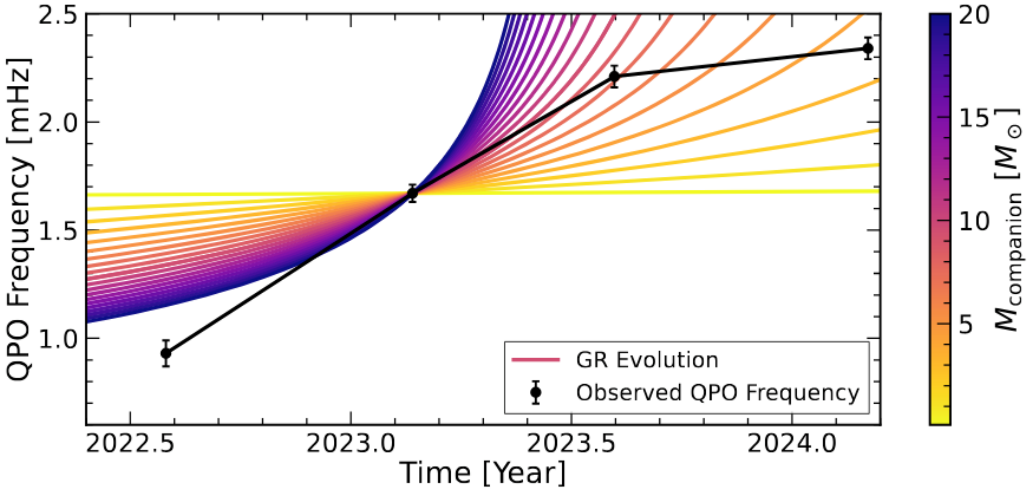
Are we seeing time-varying moments of impact as orbit and disk precess due to frame dragging?



J. Chakraborty et al., arXiv:2402.08722; ApJ **965**, 12C (2024).

Some issues

One more example: IES1927+654 (galaxy containing BH of 1.4 x 10^6 Msun) showed outburst consistent with a tidal disruption event in March 2018.



M. Masterson et al., arXiv:2501.01581; Nature **638**, 370 (2025).

Frequency evolves significantly … **NOT** in a way consistent with GW emission!!

Hypothesis: White dwarf, radiating and transferring mass onto the black hole.

If a binary, this is ***right in LISA's band***.

Relativistic EMRI waveforms are largely in hand

We can make them rapidly (fractions of a second per waveform) for a wide range of constrained systems.

Making data for generic case major focus of attention now.

Can serve as foundation for LISA science and data studies, examining post-adiabatic effects, interaction with matter …