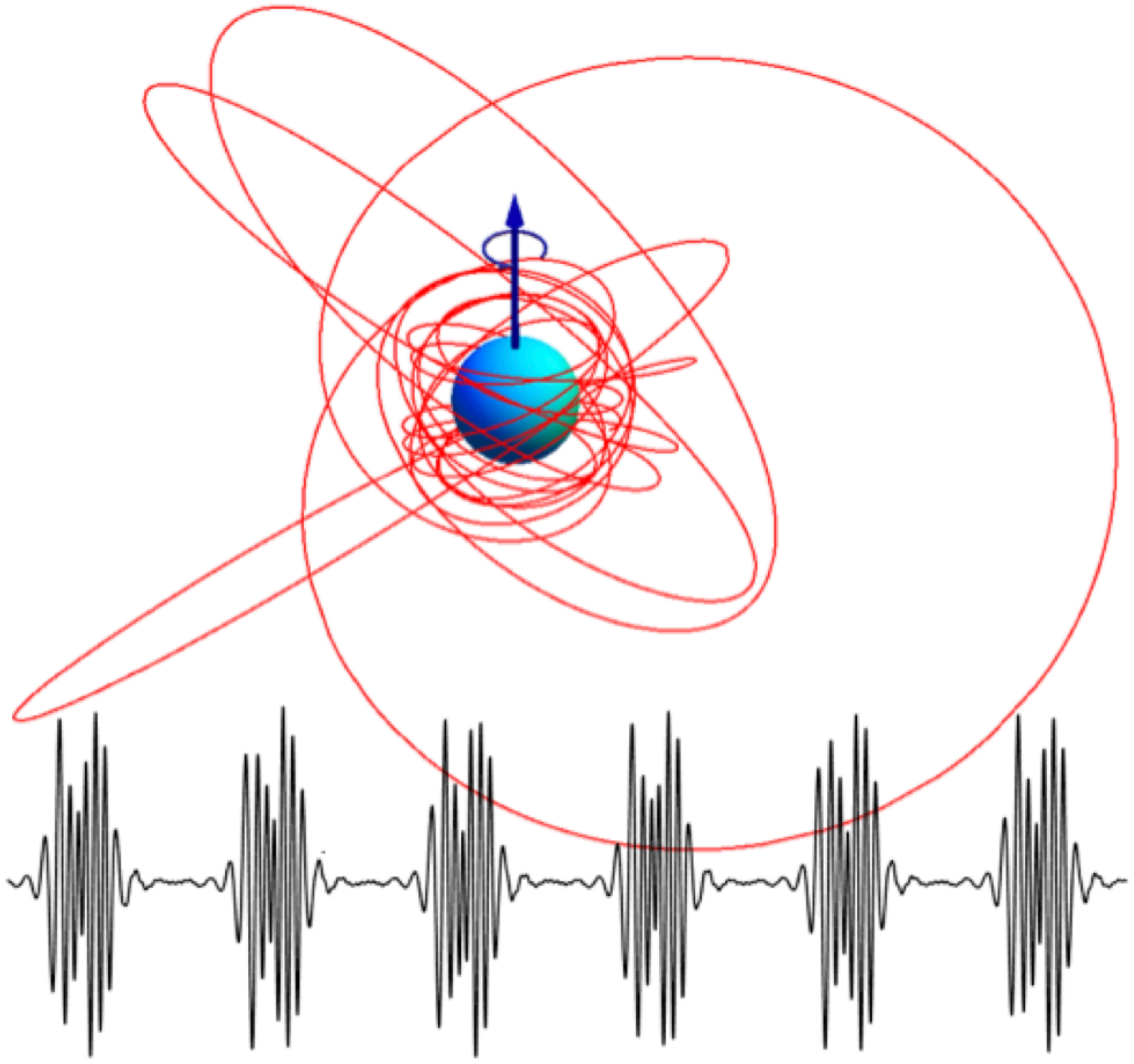
Hunting EMRIs



MONTANA STATE UNIVERSITY

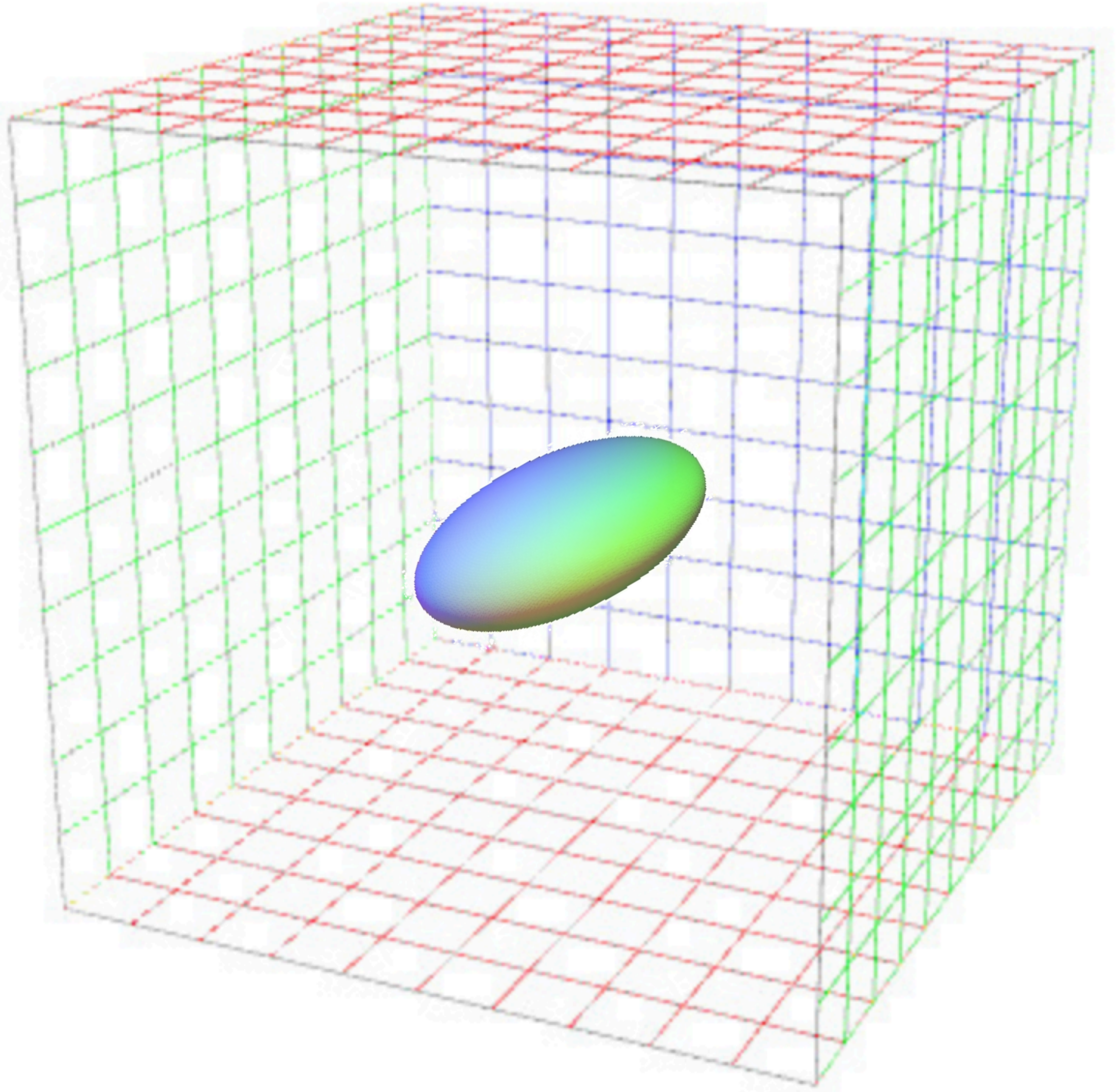
Neil J. Cornish

extreme Gravity Institute

Hunting EMRIs

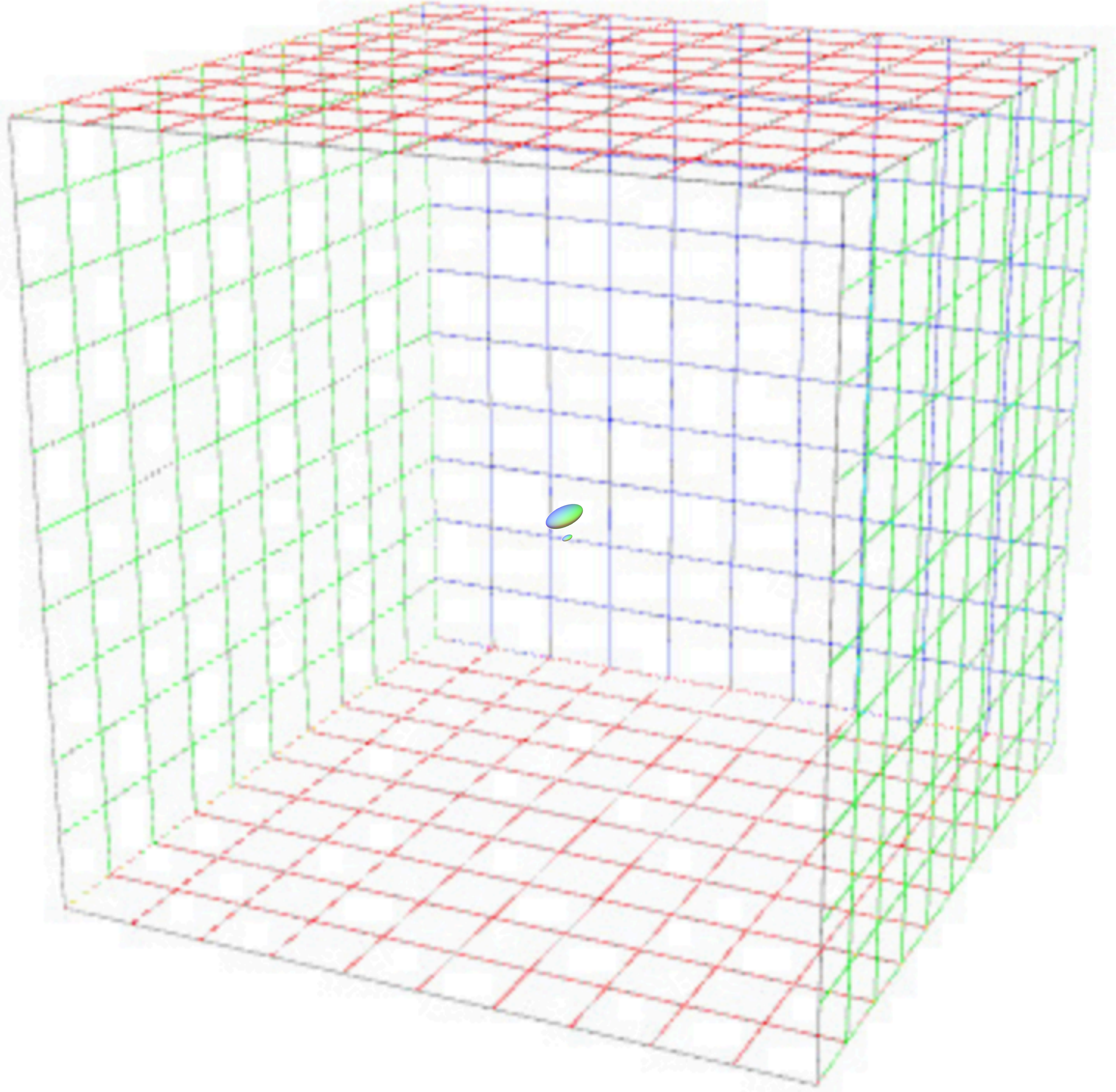
The toughest challenge in all GW science

- Why so difficult?
- Gravitational wave analysis in the mHz regime
  - Global Fit
  - Non-stationary, non-Gaussian noise + gaps
- EMRIs and the Global Fit
- Coherent, semi-coherent & hierarchical searches



Difficulty of search measured by prior to posterior volume ratio

$$\frac{\Delta V}{V}$$



Difficulty of search measured by prior to posterior volume ratio

$$\frac{\Delta V}{V} \sim 10^{-50}$$

Typical value for an EMRI

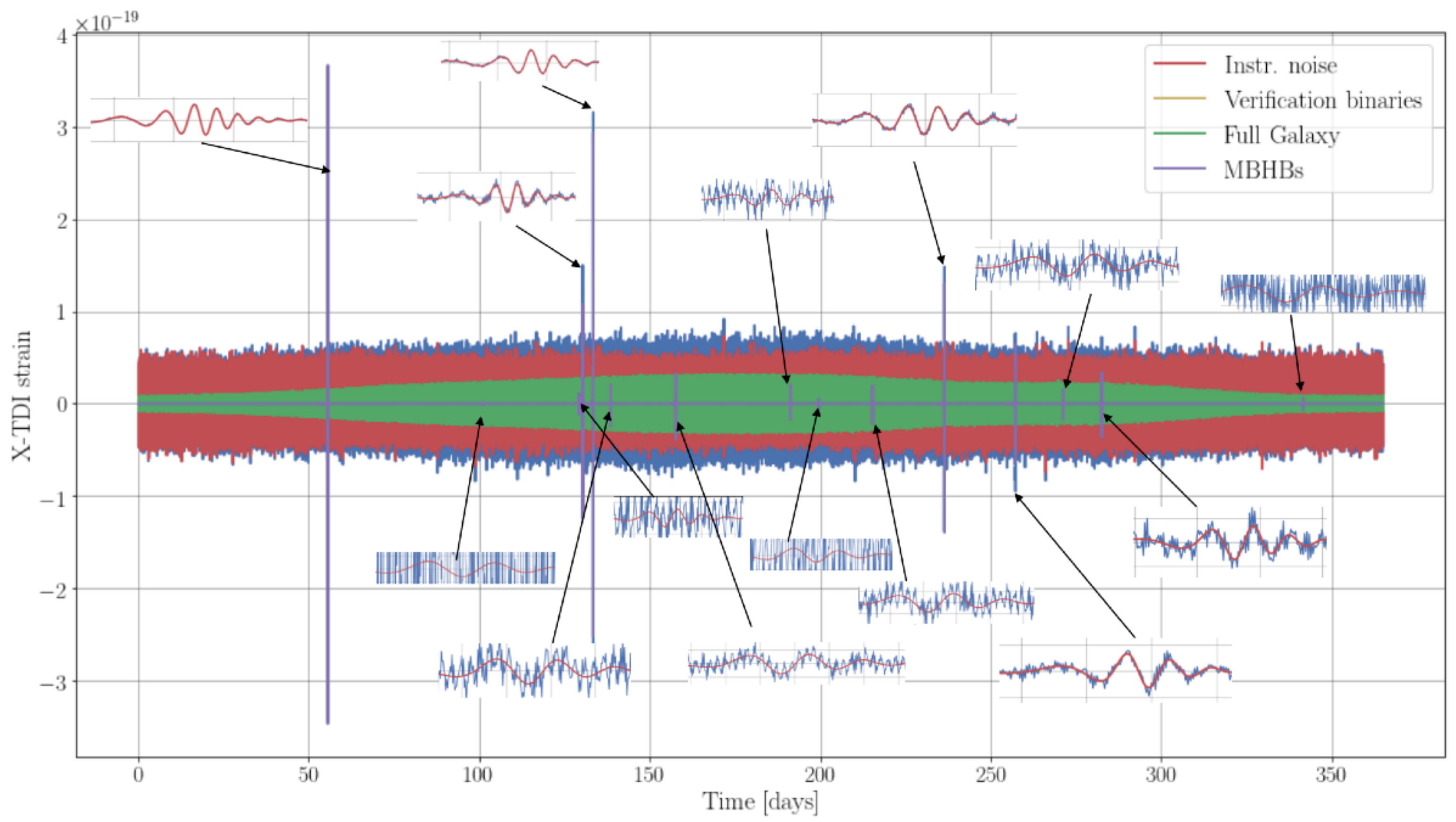
Why So Difficult?

(1) $$\frac{V_{\text{posterior}}}{V_{\text{prior}}} \sim 10^{-50}$$ Compare with $\frac{R_{\text{Proton}}}{R_{\text{Universe}}} \sim 10^{-41}$

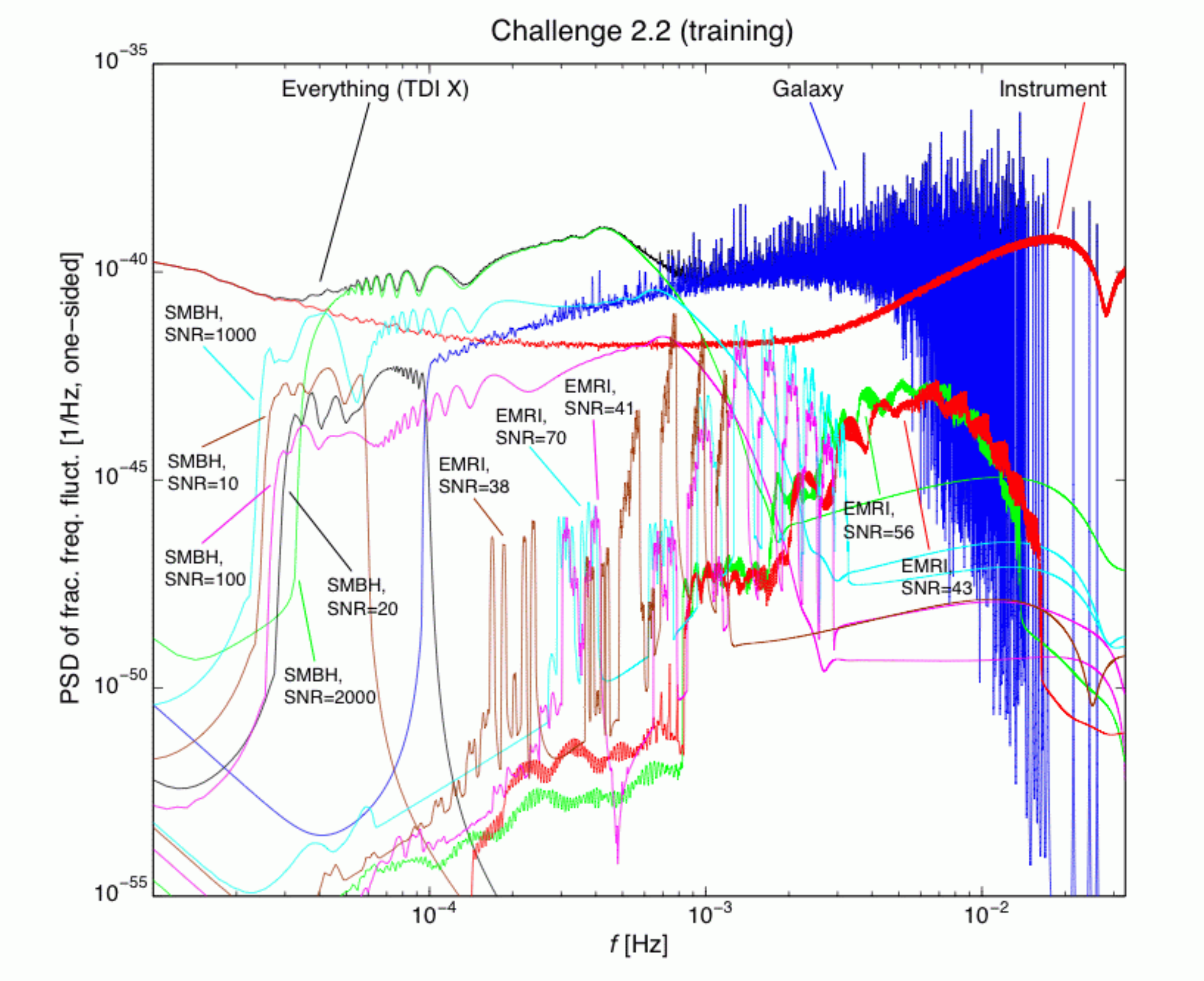
(2) Complicated waveforms, moving detector

(3) Power spread over many harmonics over many years

(4) Thousands of overlapping signals, non-stationary and non-Gaussian noise



The LISA Global Fit



$$\chi^2 = (\mathbf{d} - \mathbf{h}) \cdot \mathbf{C}^{-1} \cdot (\mathbf{d} - \mathbf{h}) = (\mathbf{d} - \mathbf{h} | \mathbf{d} - \mathbf{h})$$

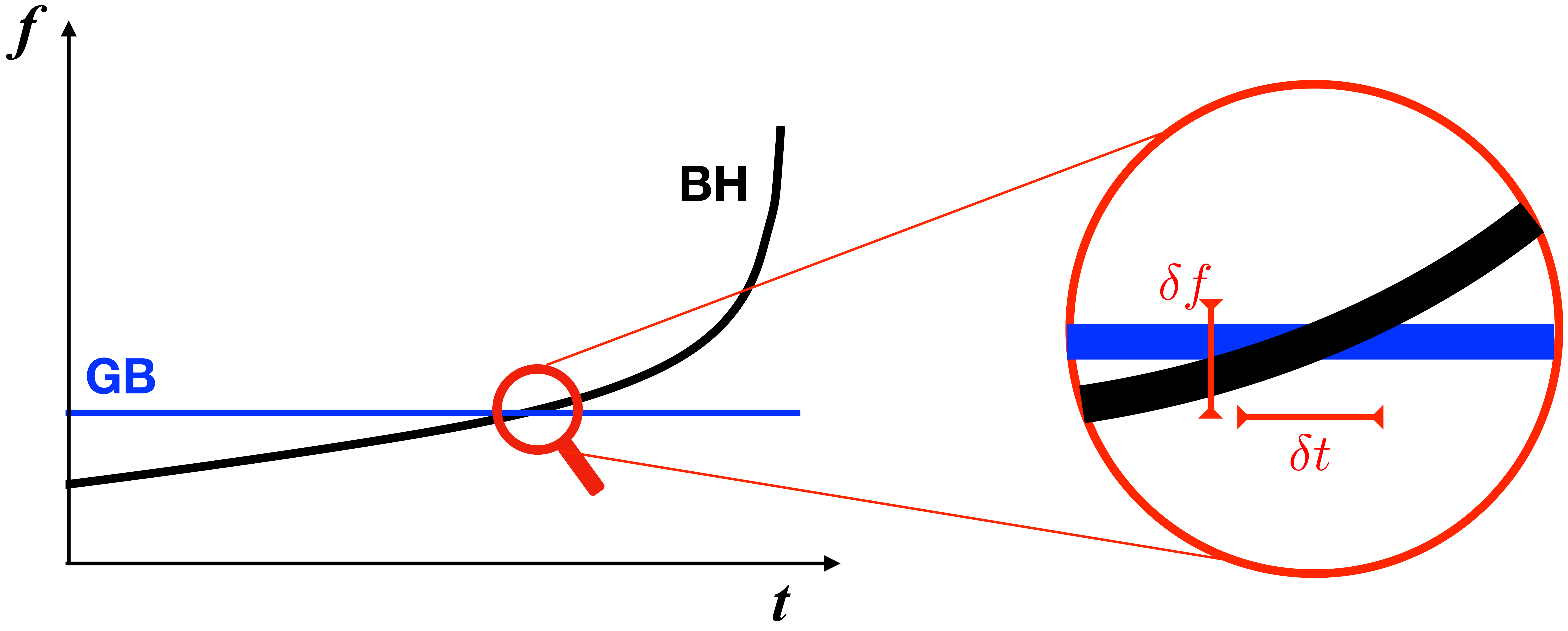
$$\chi^2 = \sum_i (\mathbf{d} - \mathbf{h}_i | \mathbf{d} - \mathbf{h}_i) + (N-1)(\mathbf{d}|\mathbf{d}) - \sum_{i \neq j} (\mathbf{h}_i | \mathbf{h}_j)$$

Individual source chi-squared

Overlap between sources

If the overlap terms were zero we wouldn't need a global solution
Individual overlaps are typically small, but there are millions of them

Signal Overlaps: Crossing the Streams

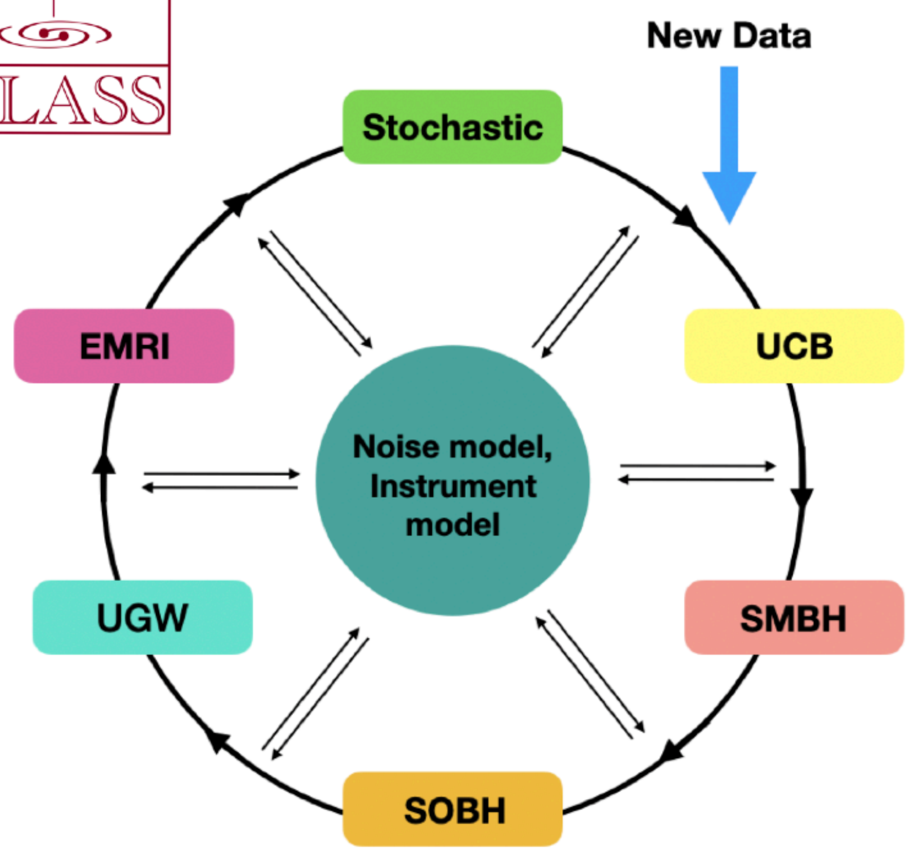


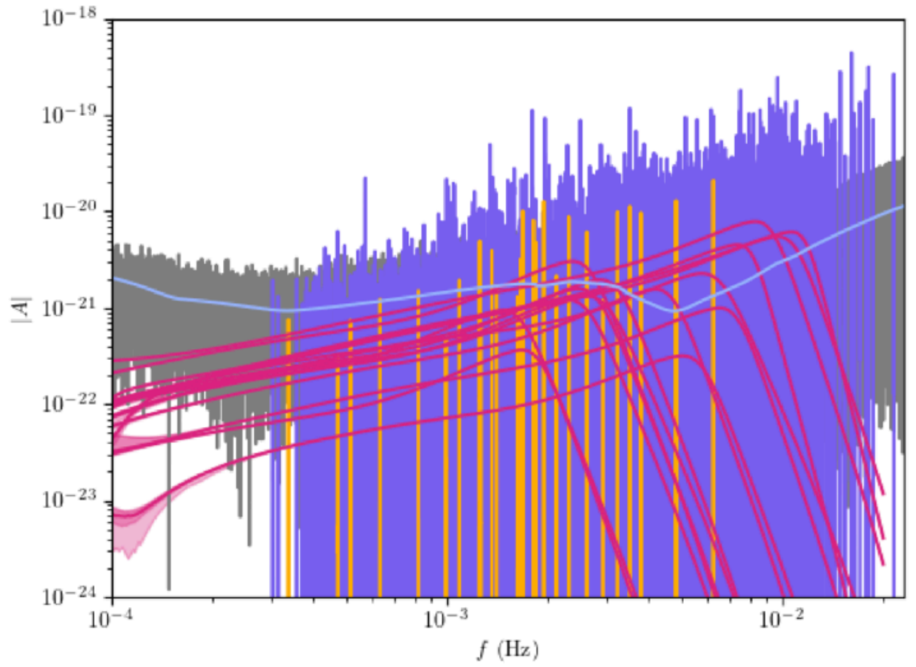
$$\delta f = 3 \times 10^{-7}\,\mathrm{mHz} \left(\frac{\mathcal{M}_{\rm GB}}{0.25 M_\odot}\right)^{5/6} \left(\frac{f_\times}{1\,\mathrm{mHz}}\right)^{11/6}$$

$$\delta t = 1.7 \times 10^{3}\,\mathrm{s} \left(\frac{10^6 M_\odot}{\mathcal{M}_{\rm BH}}\right)^{5/6} \left(\frac{1\,\mathrm{mHz}}{f_\times}\right)^{11/6}$$

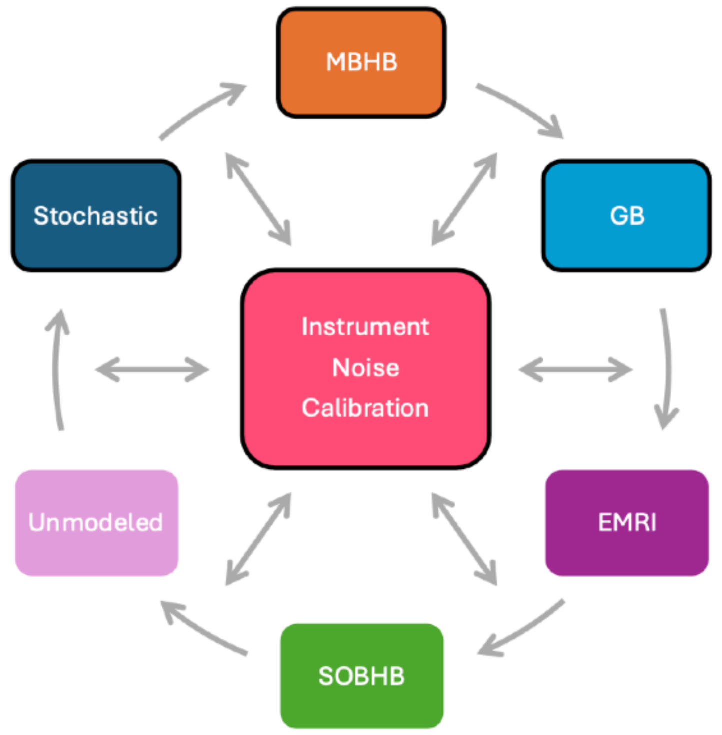
Prototype LISA Global Fits

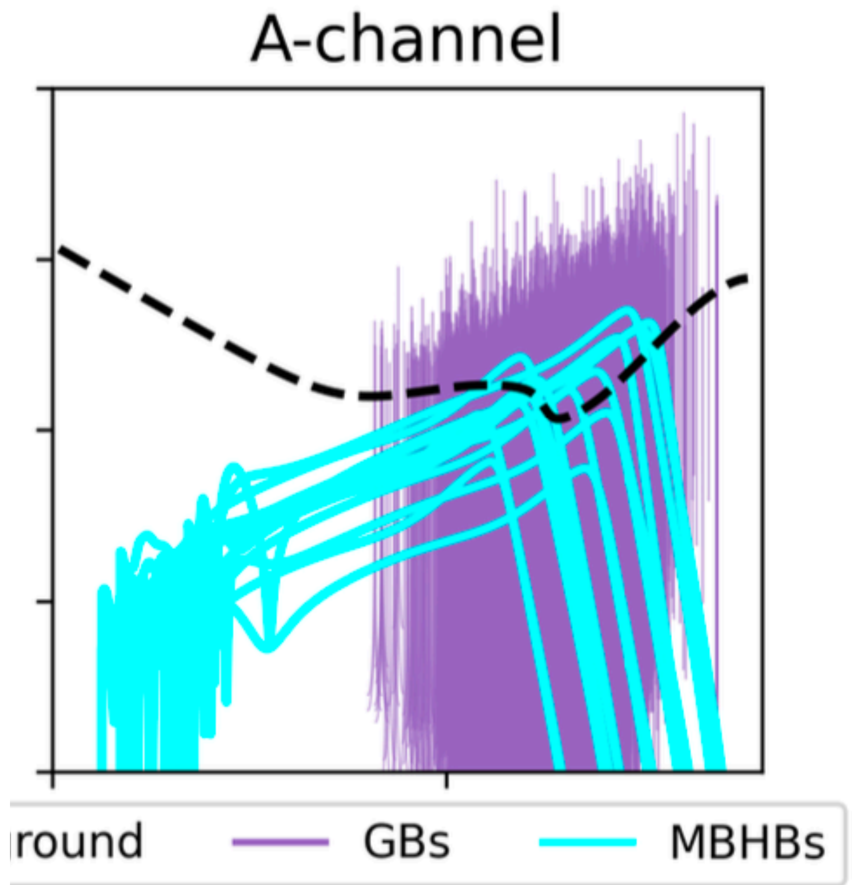
NASA/Montana: 2004.08464, 2301.03673
Fully Bayesian, Time evolving



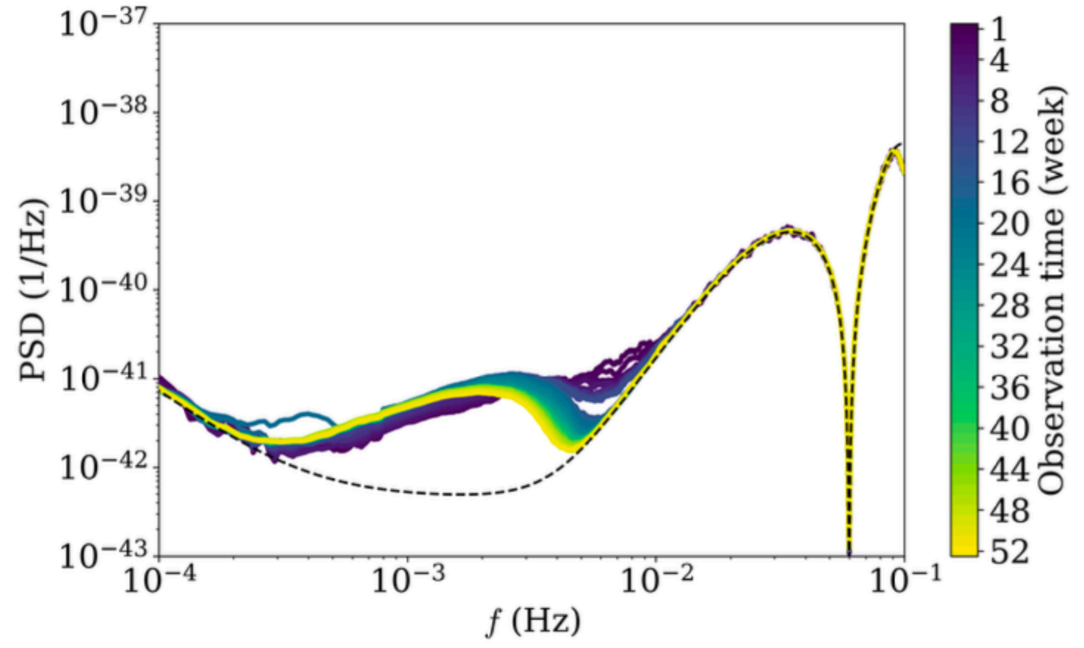


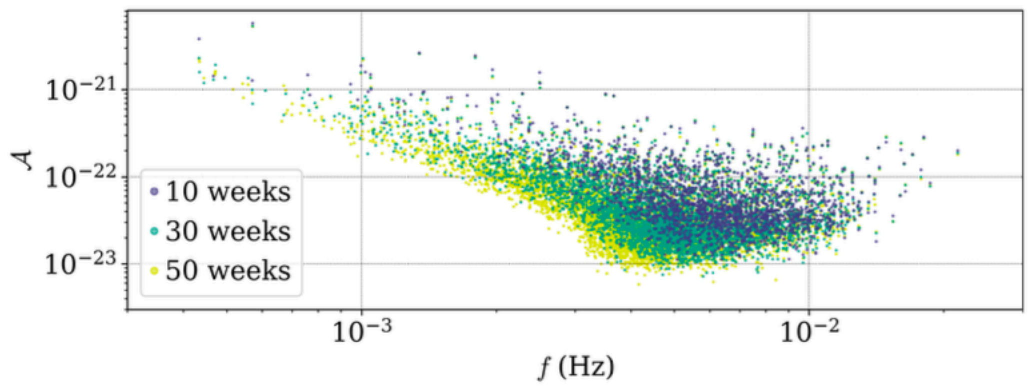
Katz et al: 2405.04690
Fully Bayesian, Non-evolving



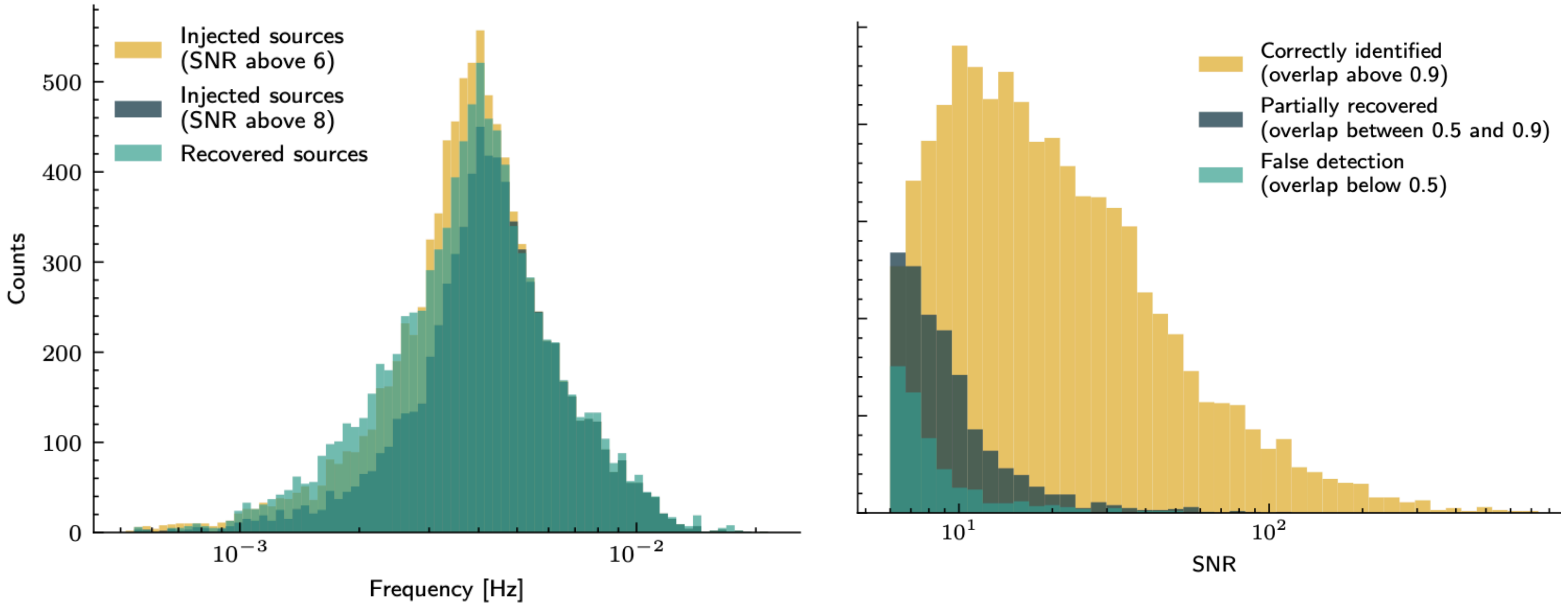


Swiss: 2403.15318, 2307.03763
Frequentist: Multi-signal ML, evolving

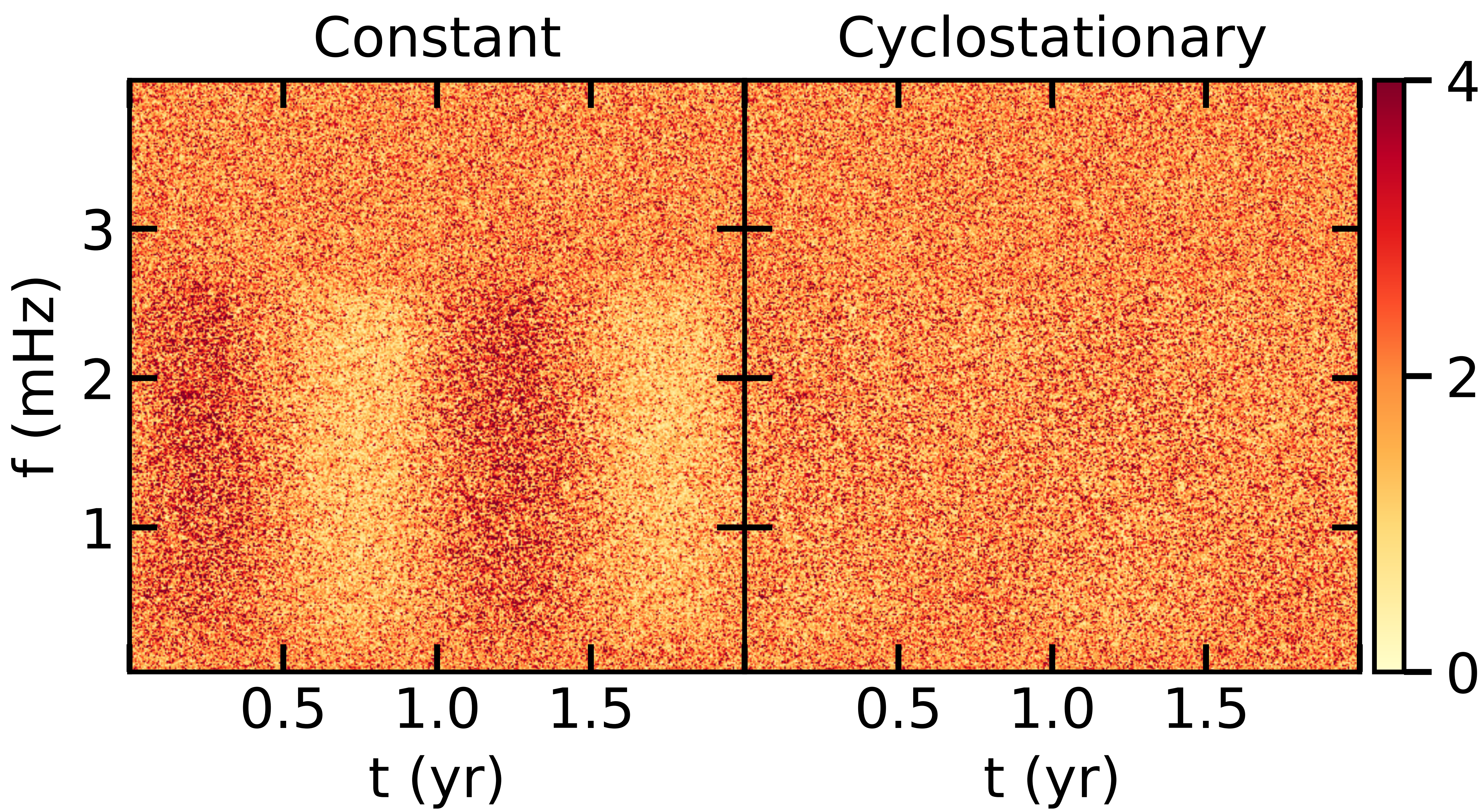




French: 2501.10277
Hybrid: Frequentist Detection
Bayesian Parameter Estimation

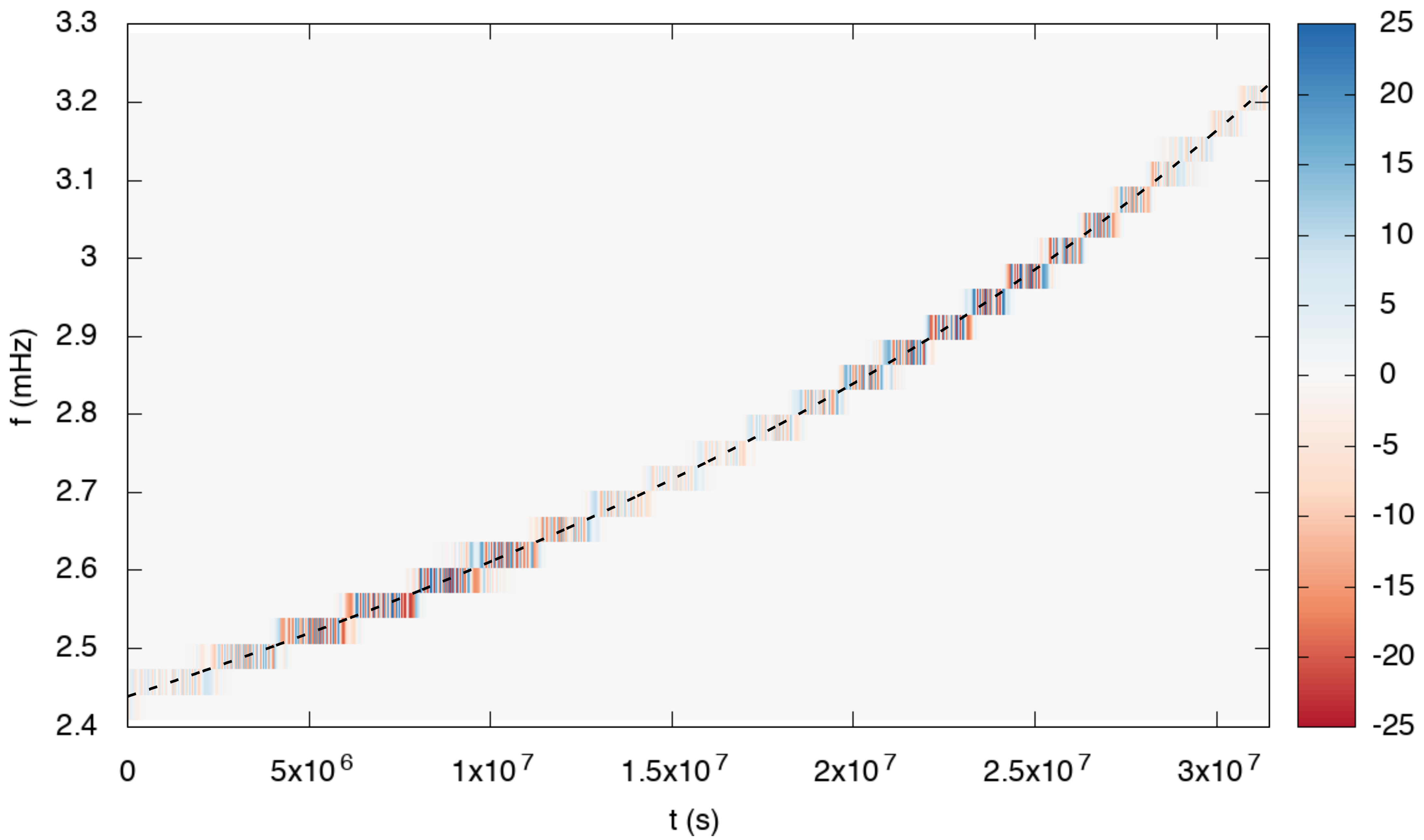


GLASS Global Fit now in the Wavelet Domain



Natural way treat non-stationary noise

[Digman & Cornish 2206.14813]



Fast wavelet transforms of the signals for computational efficiency

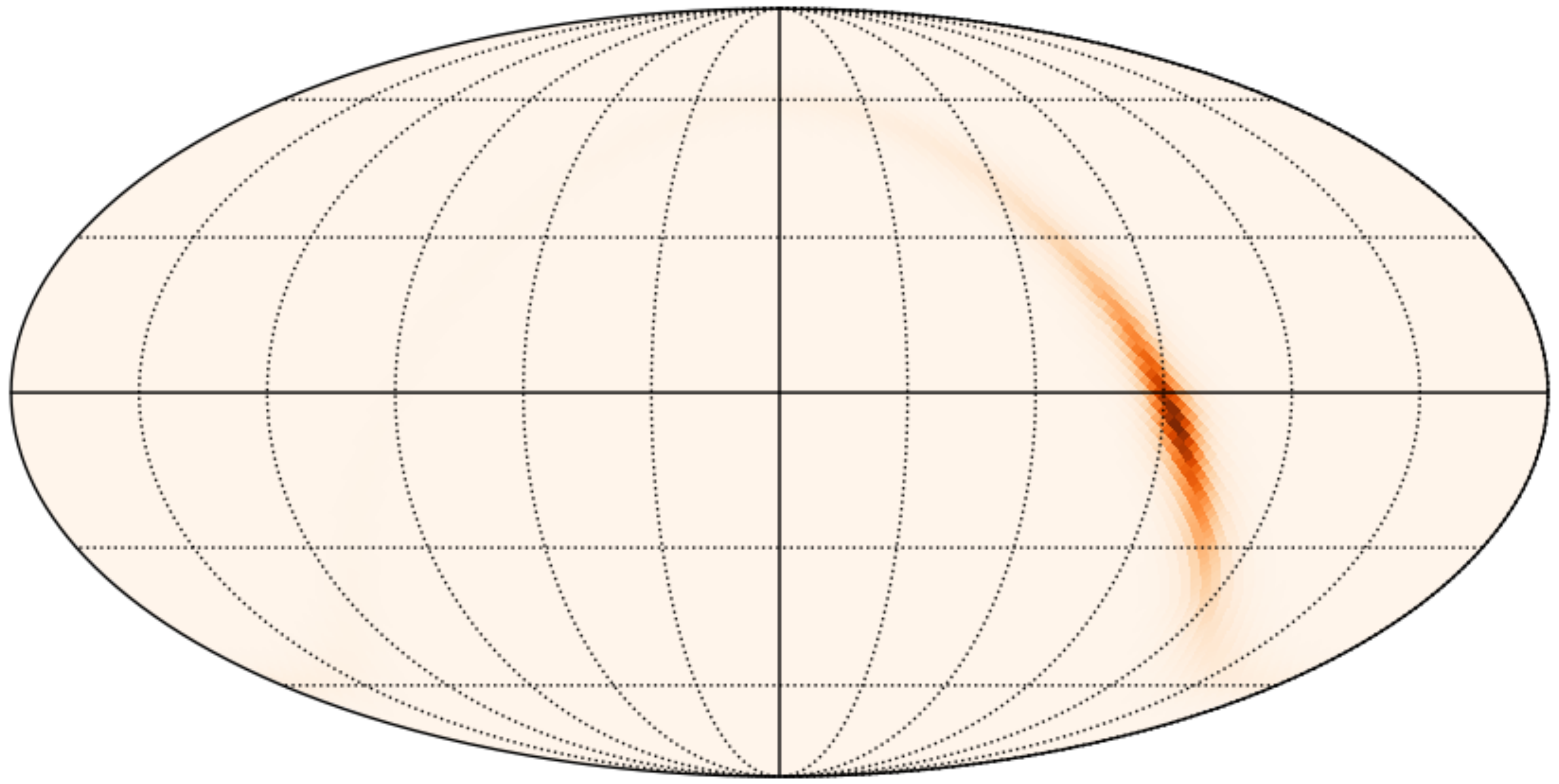
Faster than frequency domain, $\sqrt{N}$ scaling

[Cornish, Phys Rev D 102, 124038 (2020)]

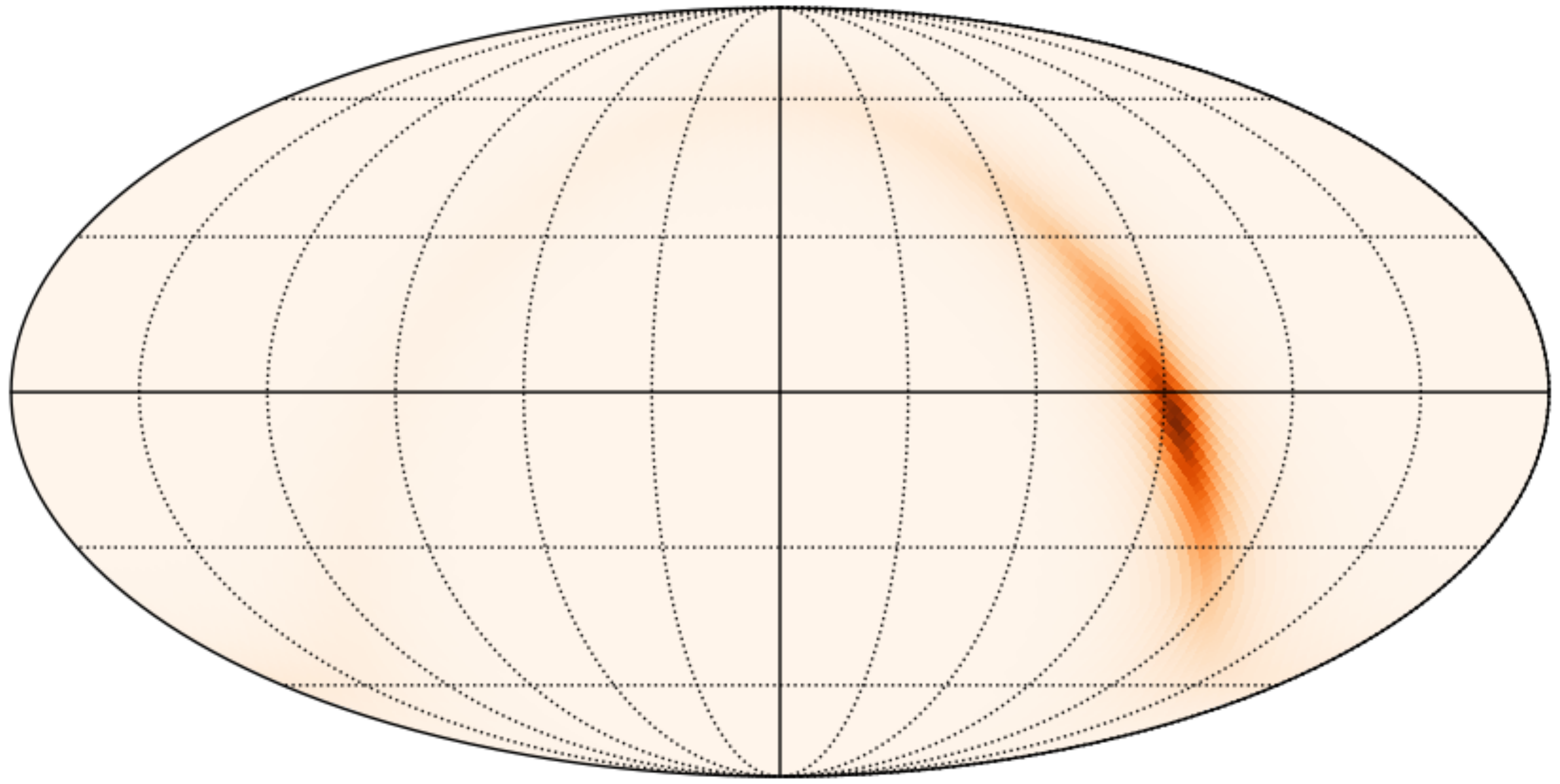
Annual Modulation of the Galactic Confusion Noise

Sudhi Mathur

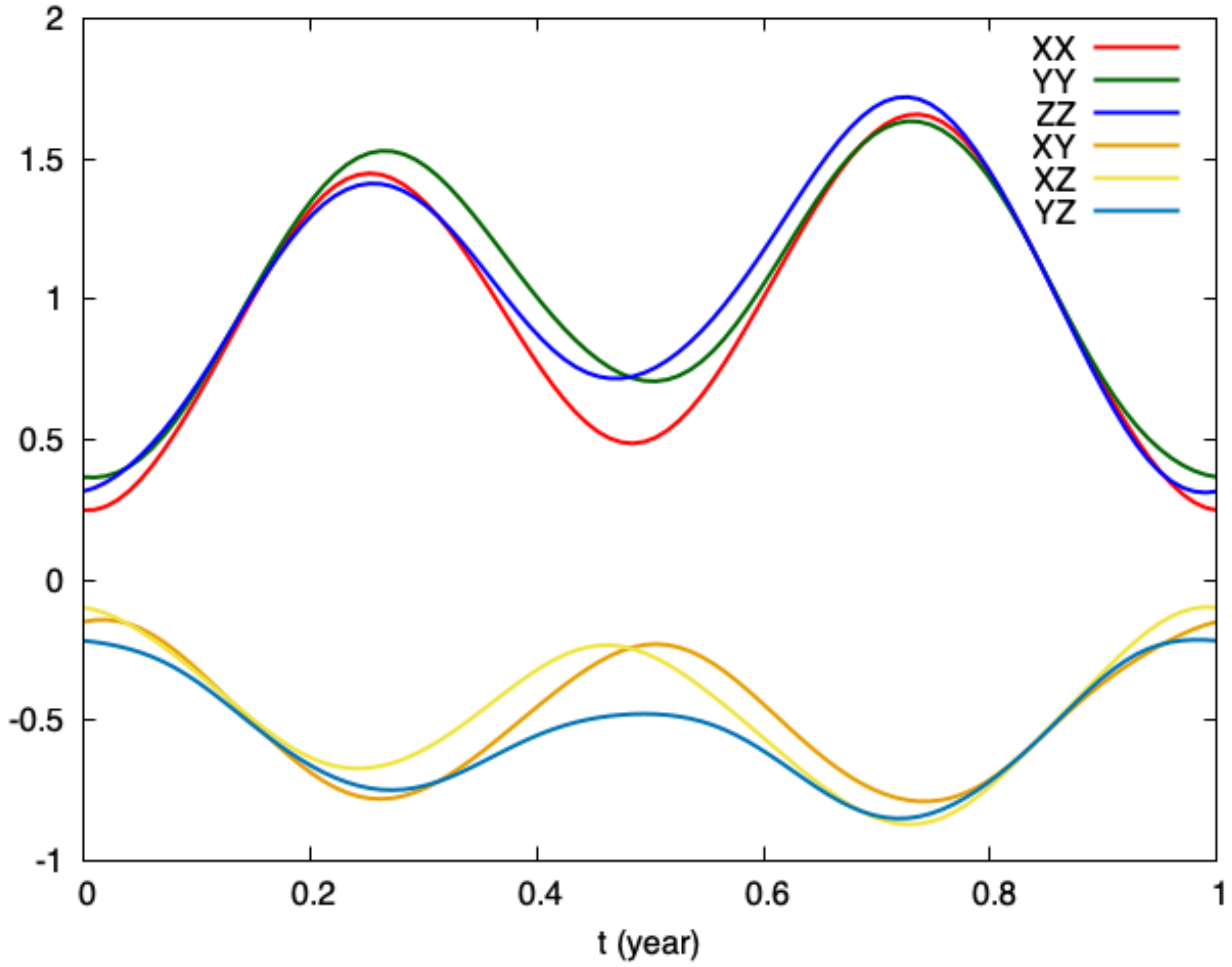
$$\rho(x,y,z) = \rho_0 \left(A e^{-r^2/R_b^2} + (1-A) e^{-u/R_d} \mathrm{sech}^2\left(\frac{z}{Z_d}\right)\right)$$



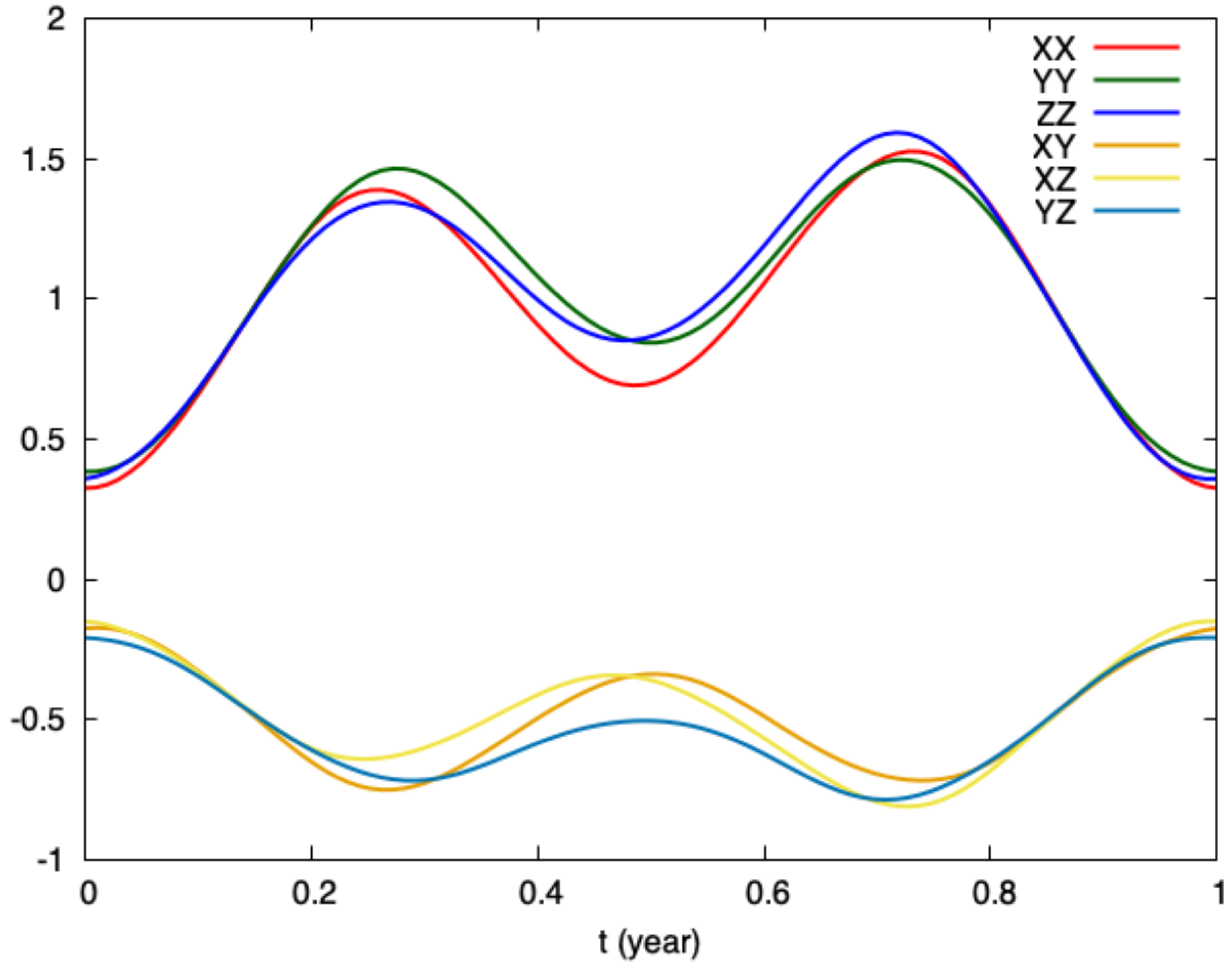
$$\mathcal{L}(\theta,\phi) \propto \int_0^\infty \rho(r,\theta,\phi)\, dr$$



$$\langle X_j X_k \rangle(t) \propto \int d\Omega\, \mathcal{L}(\theta,\phi)(D_j^{+}(\theta,\phi,t) D_k^{+}(\theta,\phi,t) + D_j^{\times}(\theta,\phi,t) D_k^{\times}(\theta,\phi,t))$$

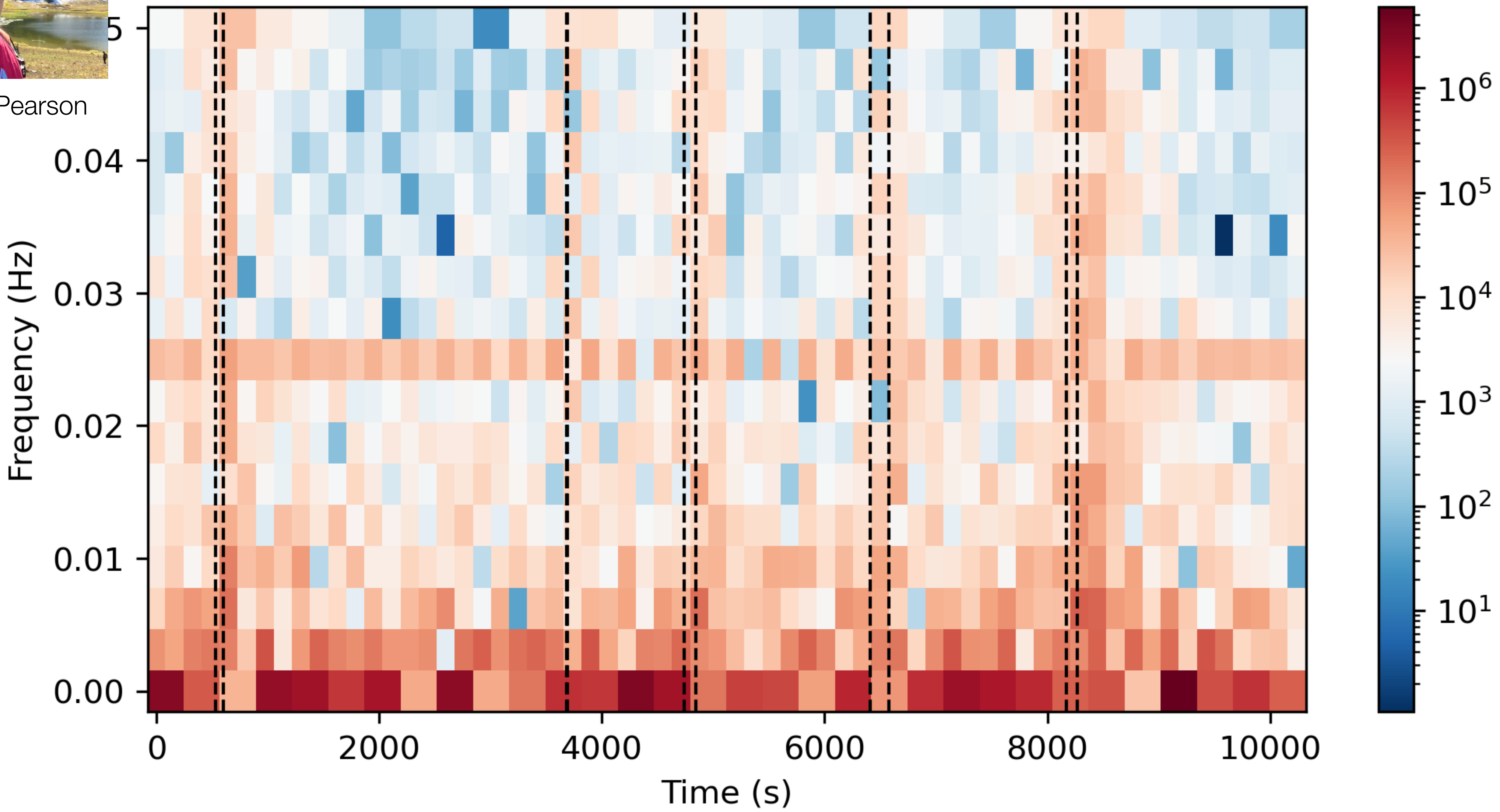


$$\langle X_j X_k \rangle(t) = \sum_{\ell=0}^{4} \sum_{m=-\ell}^{\ell} a_{\ell m}^{*}\, b_{\ell m}^{jk}(t)$$



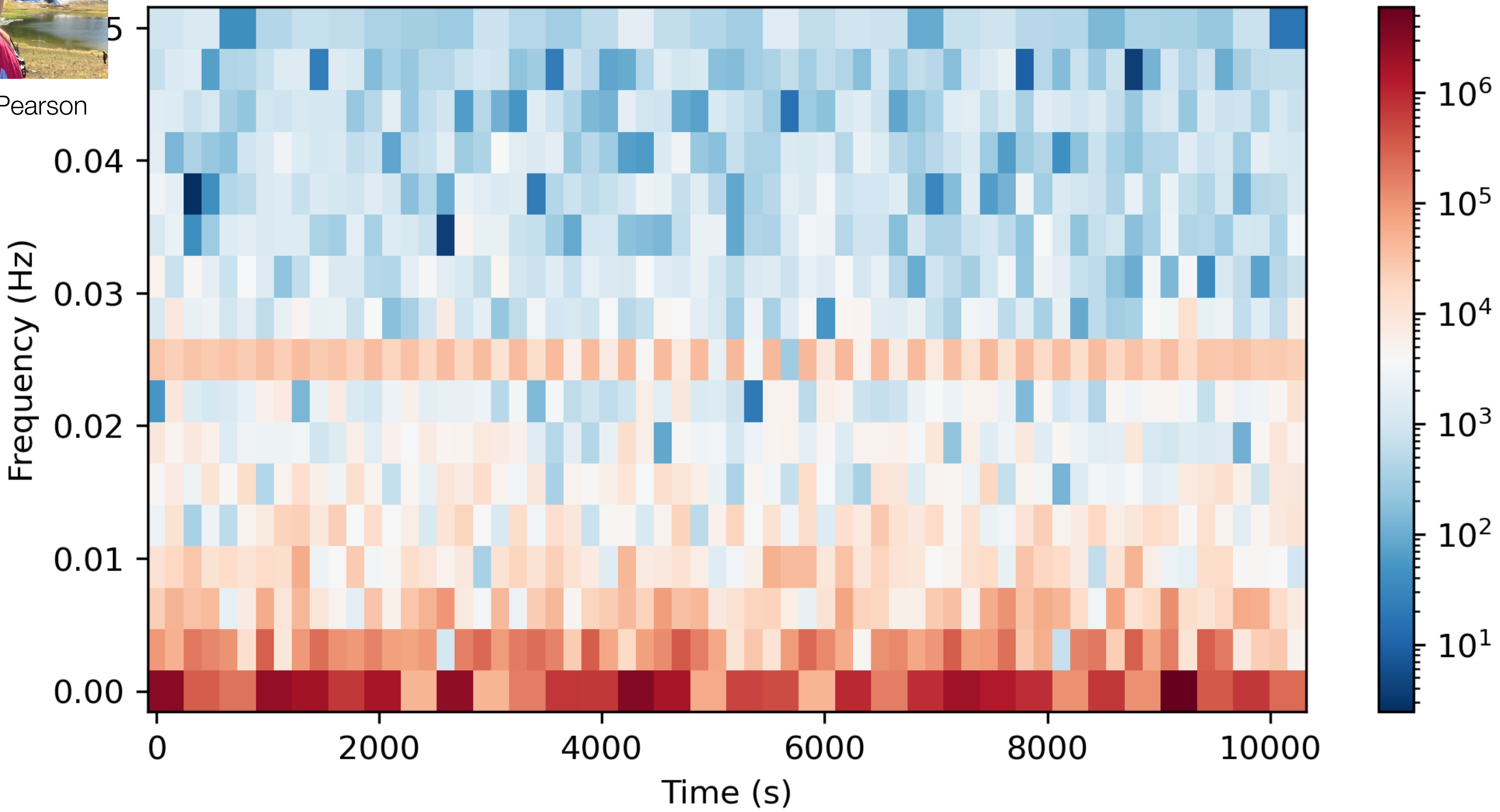
Noah Pearson

Gap Filling: Bayesian Data Augmentation



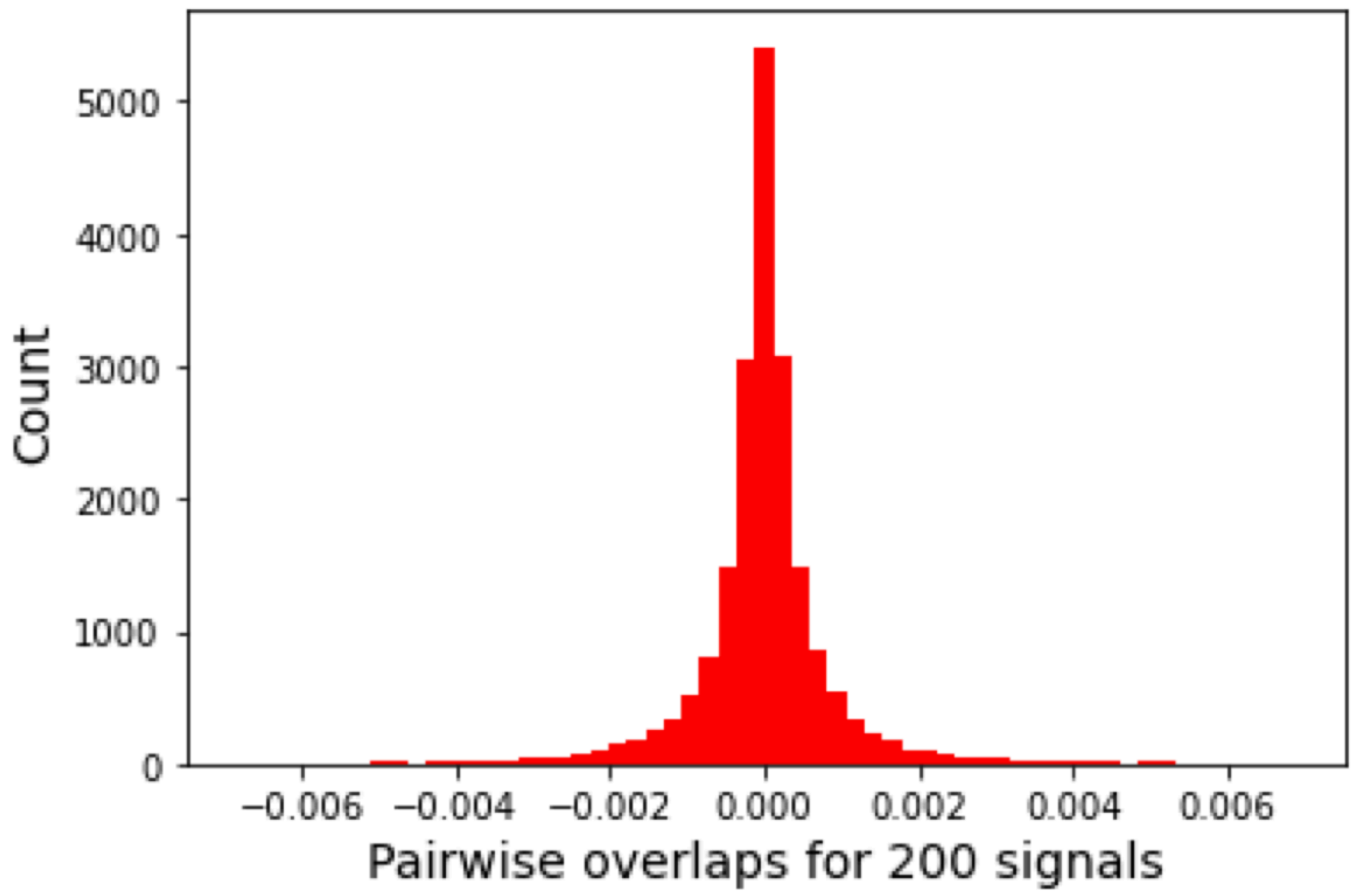
Gap Filling: Bayesian Data Augmentation

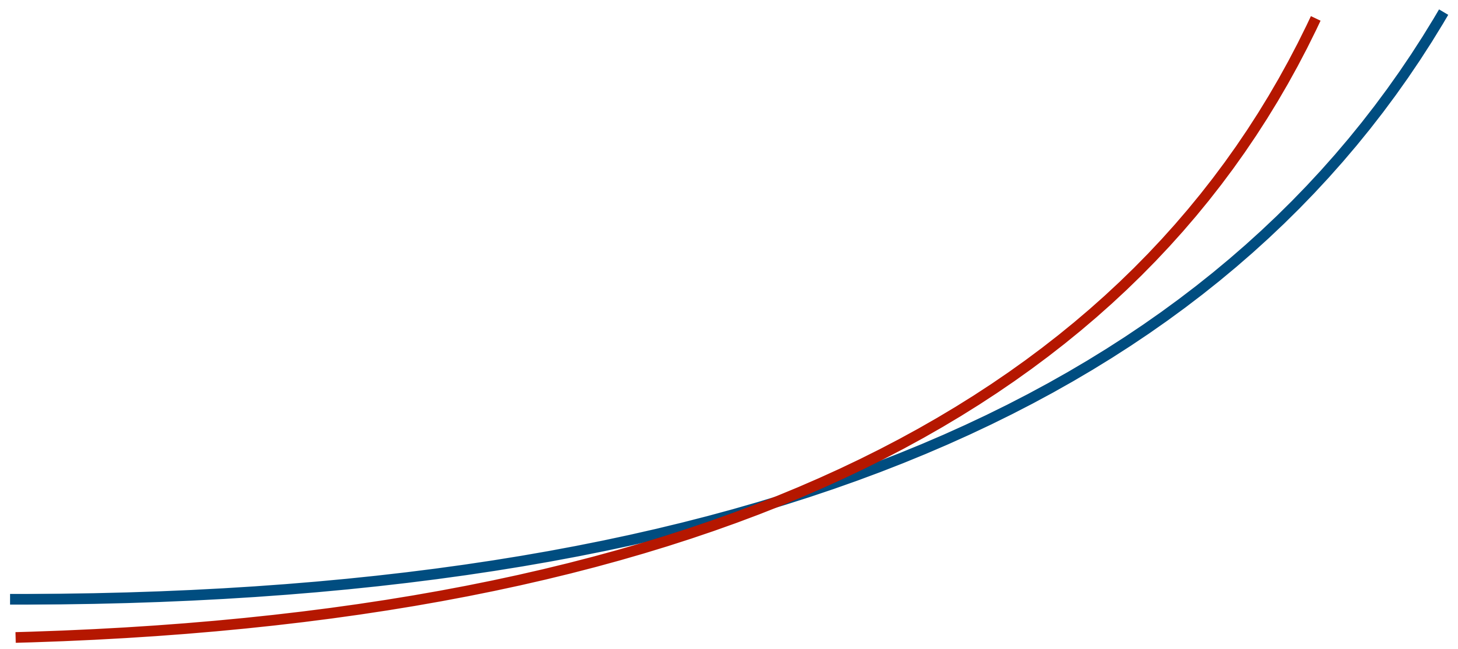
Noah Pearson



EMRIs in the Global Fit

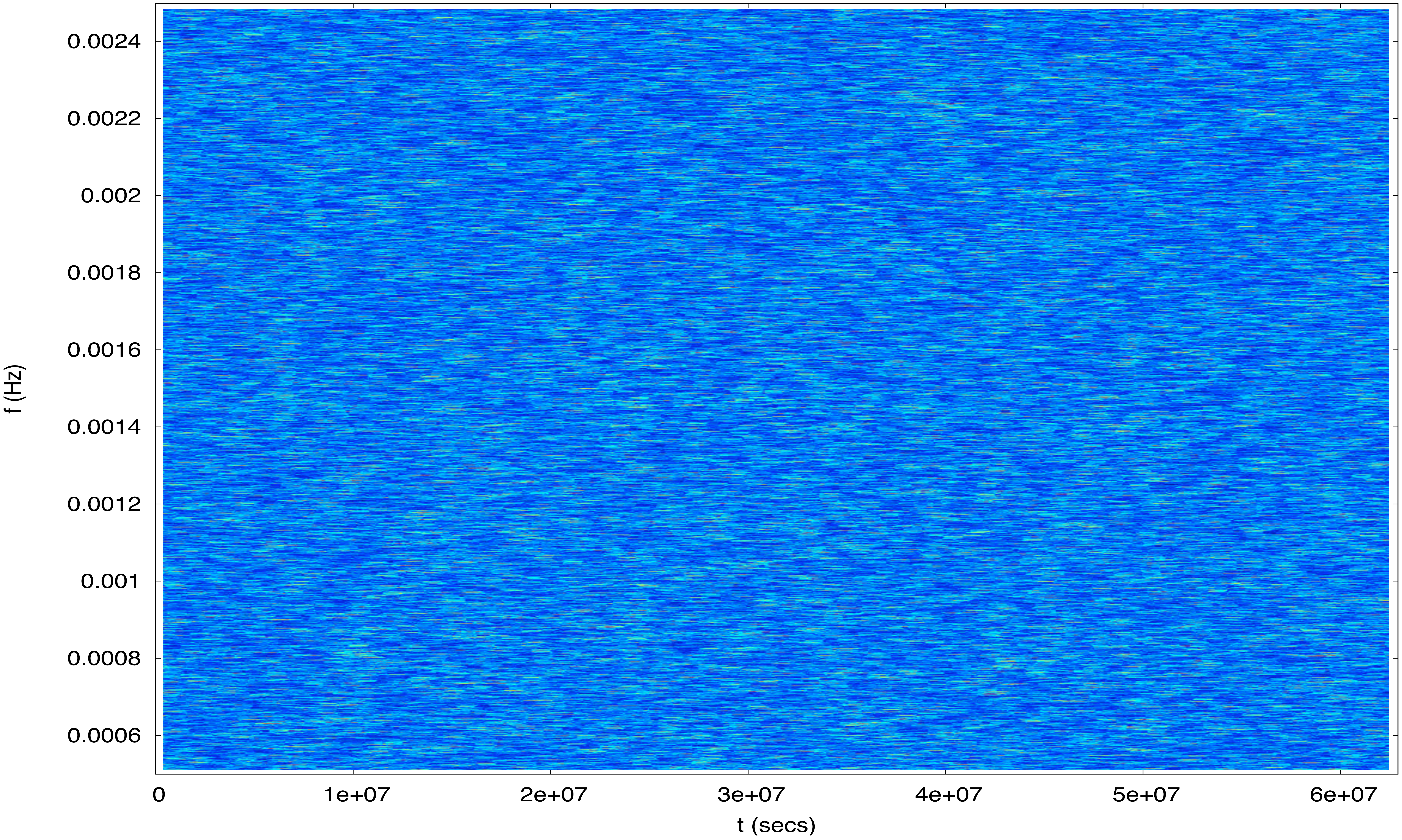
[Chua & Cutler 2109.14254]



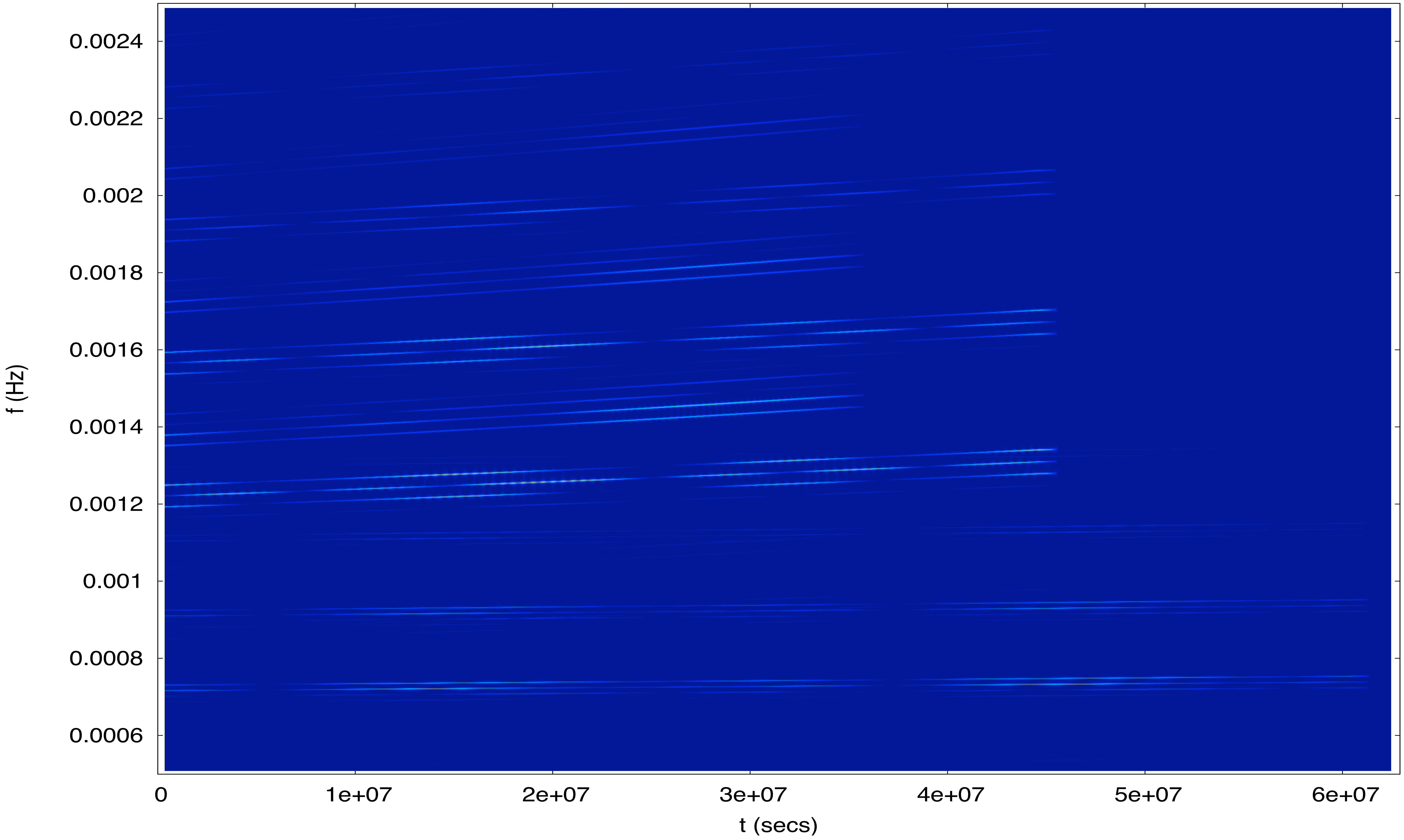


- EMRI signals are ≈ orthogonal to each other and all other signals
- Overlap depends on crossings of the t-f tracks. Some overlap for individual tracks for sources with $\mathscr{M} \sim 10^3 M_\odot - 10^4 M_\odot$
- EMRIs spread power over many harmonics and over long time times - very little power per time-frequency pixel
- Rest of the Global Fit would not be messed up if we detected zero EMRIs (unless one was ridiculously close)
- EMRIs would end up in the noise model
- Can use the Global Fit residuals to search for EMRIs
- Will need to do EMRI PE as part of Global Fit to avoid bias (marginalize over residuals and noise)

2008 MLDC - Loud EMRIs buried deep in the noise



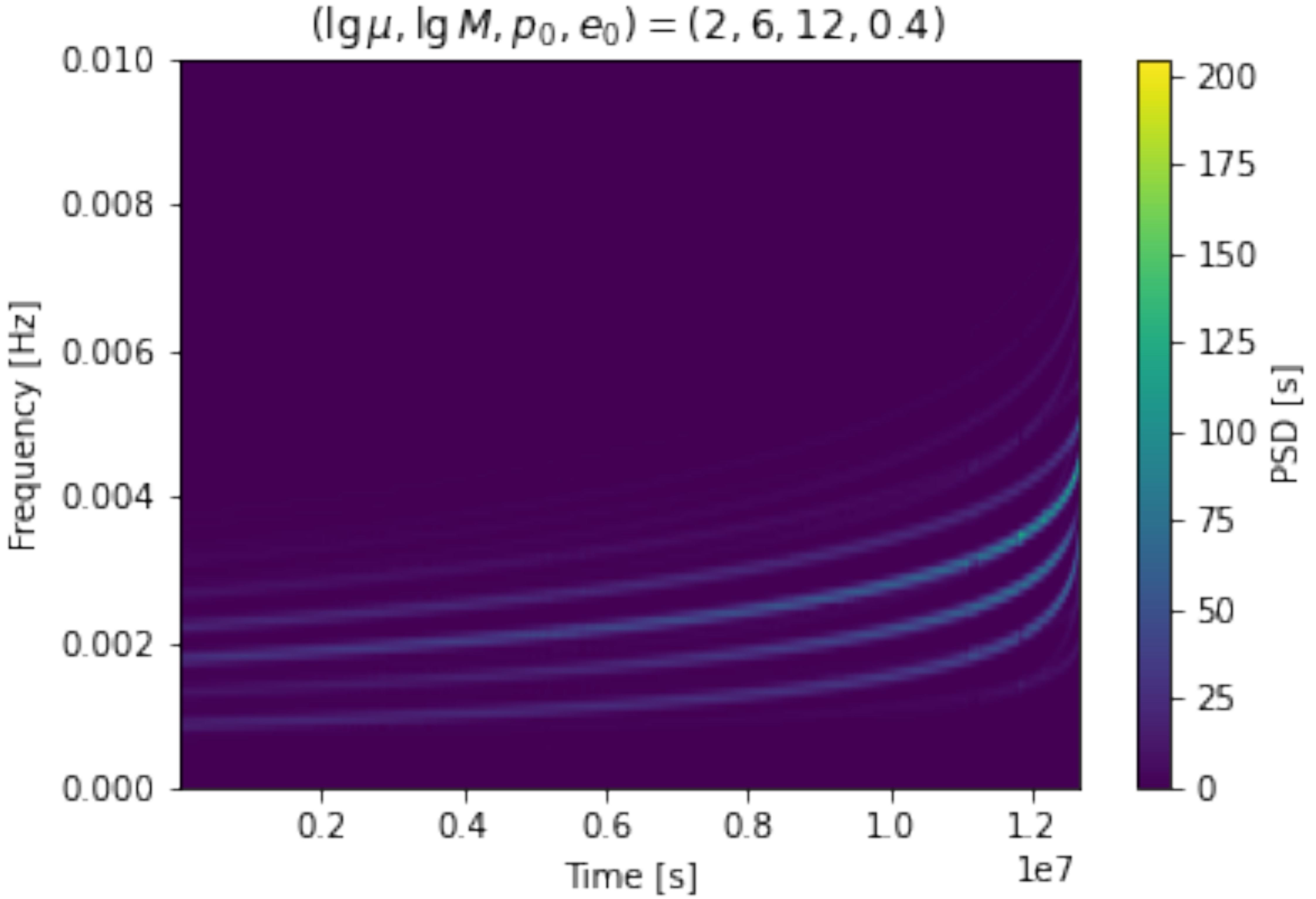
2008 MLDC - Loud EMRIs buried deep in the noise

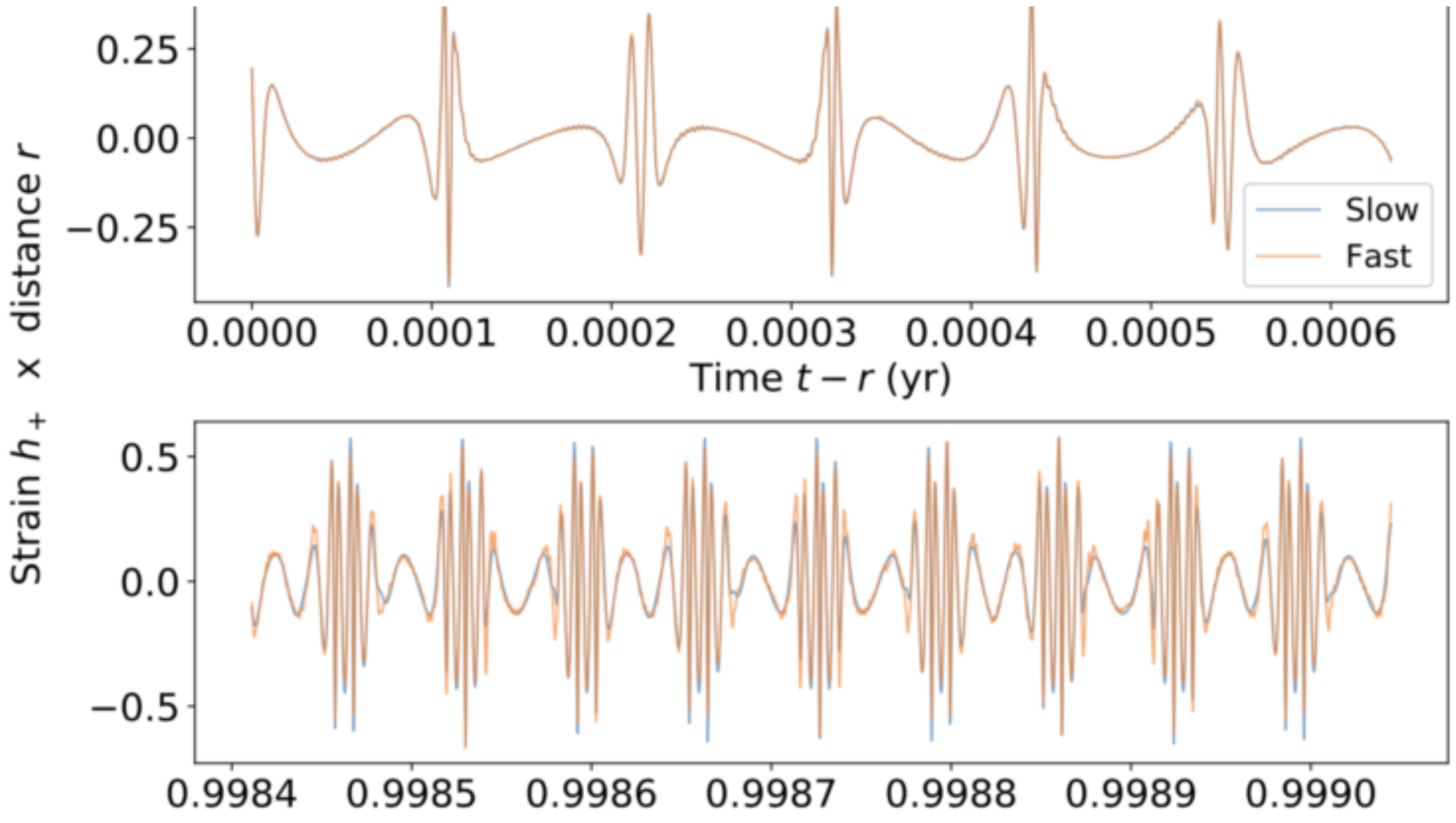


EMRI Waveforms - Voices

$$h_+ - ih_\times = \frac{\mu}{D_L} \sum_{lmkn} A_{lmkn}(t) S_{lmkn}(t,\theta) e^{im\phi} e^{-i\Phi_{mkn}}$$

Fast wavelet domain versions are very natural





[Katz, Chua, Speri, Warburton & Hughes, arXiv:2104.04582]

Hunting EMRIs - General Principles

- An optimal search is computationally infeasible (unless you have a large quantum computer)
- Use semi-coherent and hierarchical methods
- Analytically maximize over as many parameters as possible in the likelihood (softens the target)
- Have to contend with many secondary maxima - blessing or curse?
- Phenomenological models may be sufficient for detection
- The last EMRI will cost as much to detect as the total for all the EMRIs that came before

Coherent searches are costly

Need long integration times to capture enough Signal-to-noise squared: $\mathrm{SNR}^2 \sim T$

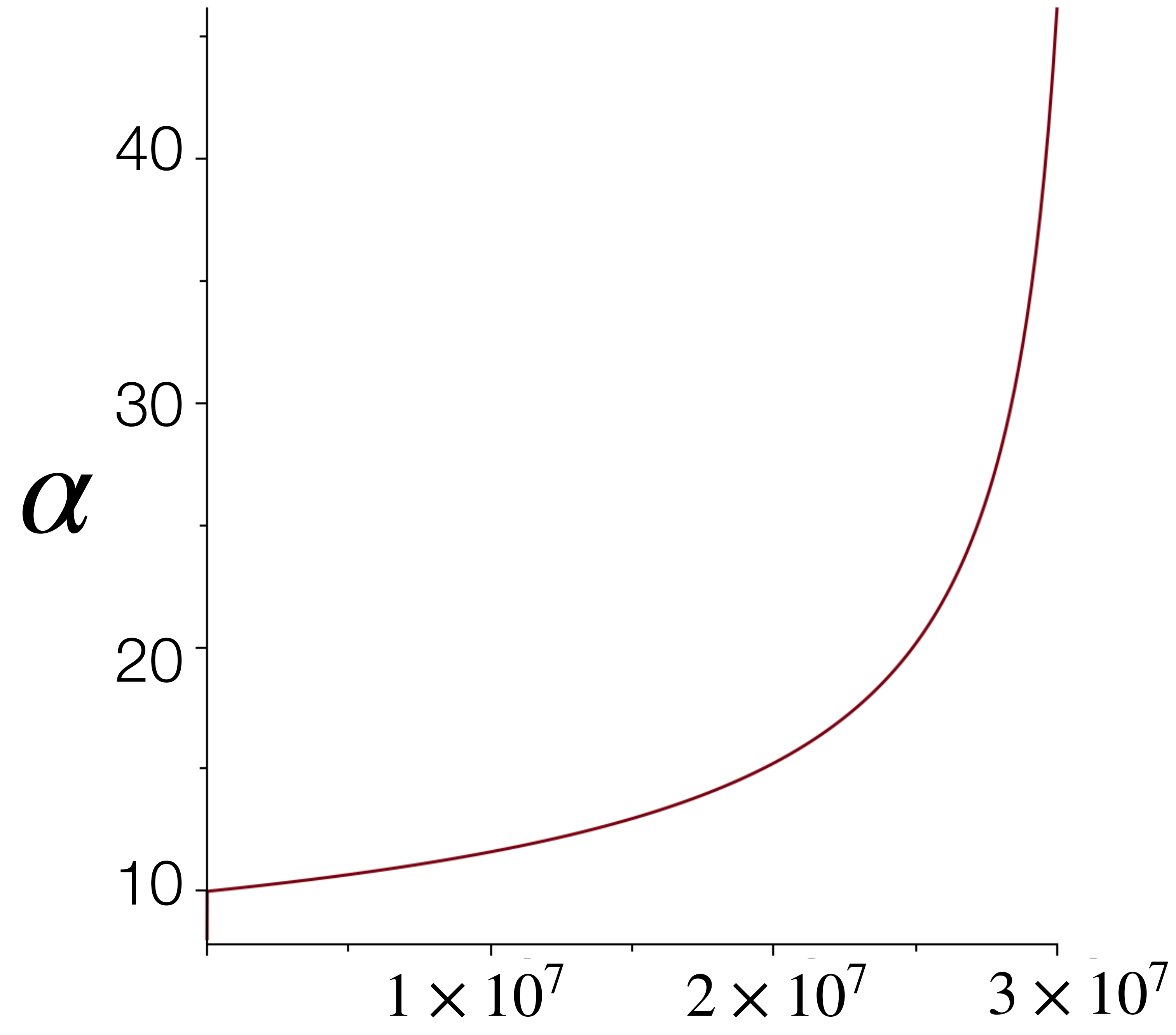
But the search cost increases rapidly with integration time….

Quadratic Chirp $$h(t) = A\cos(2\pi f_0 t + \pi \dot{f}_0 t^2 + \pi/3 \ddot{f}_0 t^3 + \phi_0)$$

Prior/Posterior volume ratio $$\frac{\Delta V}{V} \sim T^{-8.5}$$

0PN Chirp $$h(t) = A\left(\frac{5\mathcal{M}}{t_c - t}\right)^{1/4} \cos\left[\phi_c - 2\left(\frac{5\mathcal{M}}{t_c - t}\right)^{-5/8}\right]$$

Prior/Posterior volume ratio $$\frac{\Delta V}{V} \sim T^{-\alpha}$$



Semi-coherent searches

A semi-coherent search breaks up the analysis into N short segments $T_{\rm coh}$ with $T = N\,T_{\rm coh}$

$$\text{cost} \sim T^m\, T_{\rm coh}^{n-m} \qquad\qquad m \sim 2, \quad n \gg m$$

Semi-coherent searches can be much cheaper than fully coherent searches. But they are less sensitive

Minimum detectable amplitude for a coherent search $$h_{\rm min} \sim \frac{1}{T^{1/2}}$$

Minimum detectable amplitude for a semi-coherent search $$h_{\rm min} \sim \frac{1}{T^{1/4}\, T_{\rm coh}^{1/4}}$$

Semi-coherent searches

A semi-coherent search breaks up the analysis into N short segments $T_{\rm coh}$ with $T = N\, T_{\rm coh}$

$$\text{cost} \sim T^m\, T_{\rm coh}^{n-m} \qquad\qquad m \sim 2, \quad n \gg m$$

Semi-coherent searches can be much cheaper than fully coherent searches. But they are less sensitive

Minimum detectable amplitude for a coherent search $\qquad h_{\rm min} \sim \dfrac{1}{T^{1/2}}$

Minimum detectable amplitude for a semi-coherent search $\qquad h_{\rm min} \sim \dfrac{1}{T^{1/4}\, T_{\rm coh}^{1/4}}$

But we don't need to unambiguously detect signals with the hierarchical search - just need candidates for coherent follow up

Kallol Dey

A new type of semi-coherent search

[Dey & Cornish in progress]

- Analytically maximize over as many parameters as possible in the likelihood (softens the target)
- Use greedy Monte Carlo sampling - many walkers, inverted parallel tempering, gradient information (Langevin MCMC). Semi Markovian
- Many posteriors modes for each segment searched (including peaks due to noise). Product of posteriors reinforces valid signals. Finish with fully coherent likelihood

Maximizing over extrinsic parameters - Generalized F-statistic

Simplest maximization is over overall amplitude and phase - just need two quadrature filters

F-statistic was originally derived for binary systems at quadrupole order with zero eccentricity and spin. Using four filter functions could analytically maximize over four extrinsic parameters (amplitude, phase, inclination, polarization)

Can be generalized to cover generic spin-precessing binaries and moving detectors. Just need more filters.

Example $\ell = 2$ mode, 5 harmonics. Need 40 filters

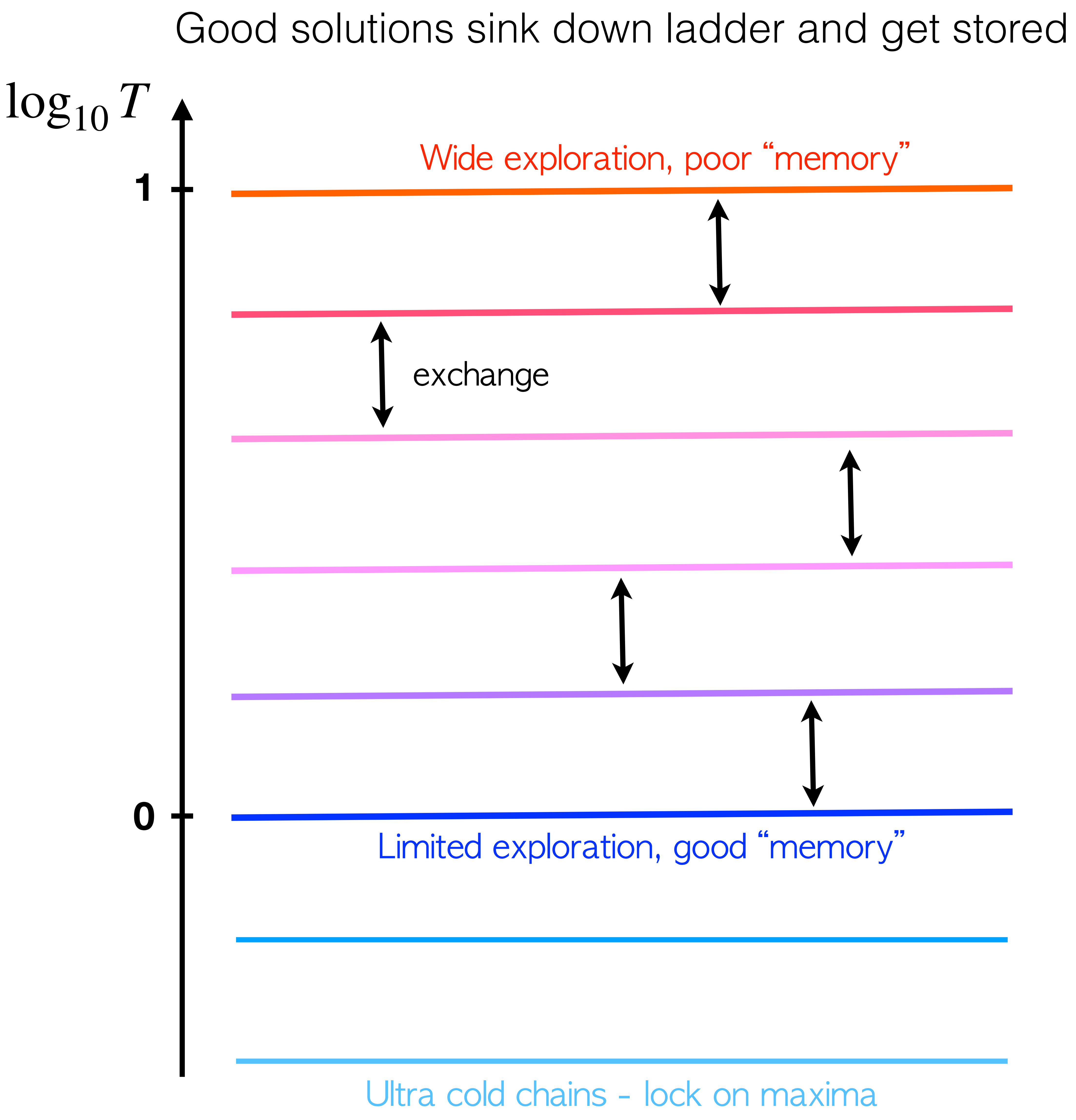
$$h(t) = \sum_{k=0}^{4} \left(\mathscr{A}_k^1 h_{0,+}^k(t) + \mathscr{A}_k^3 h_{\pi/2,+}^k(t) + \mathscr{A}_k^2 h_{0,\times}^k(t) + \mathscr{A}_k^4 h_{\pi/2,\times}^k(t) \right)$$

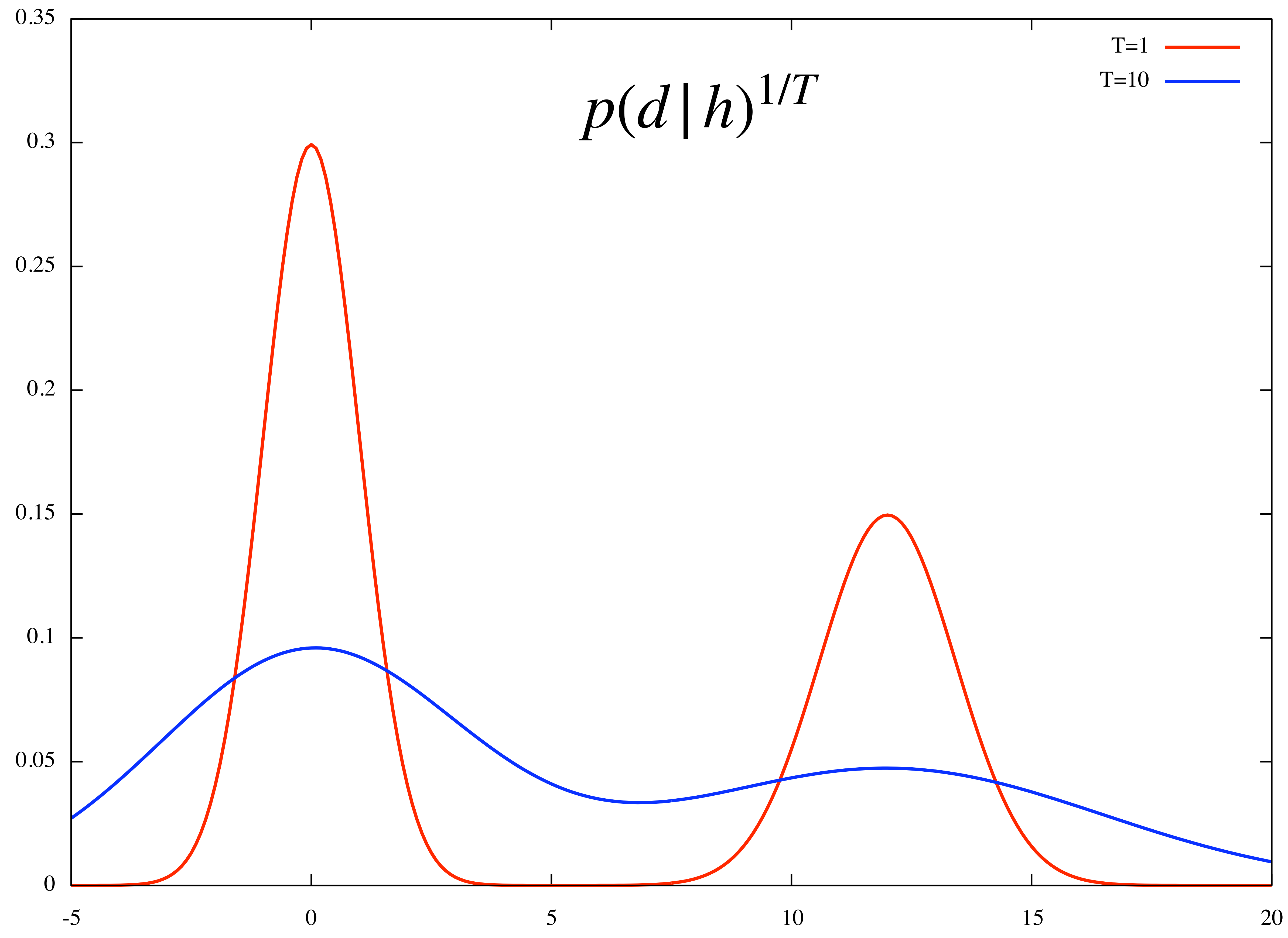
The amplitudes $\mathscr{A}_k^j$ are constant, and depend on the overall amplitude scaling and phase, and the relative orientation of the detector frame and source frame.

The time dependent filters fold in the time dependent instrument response

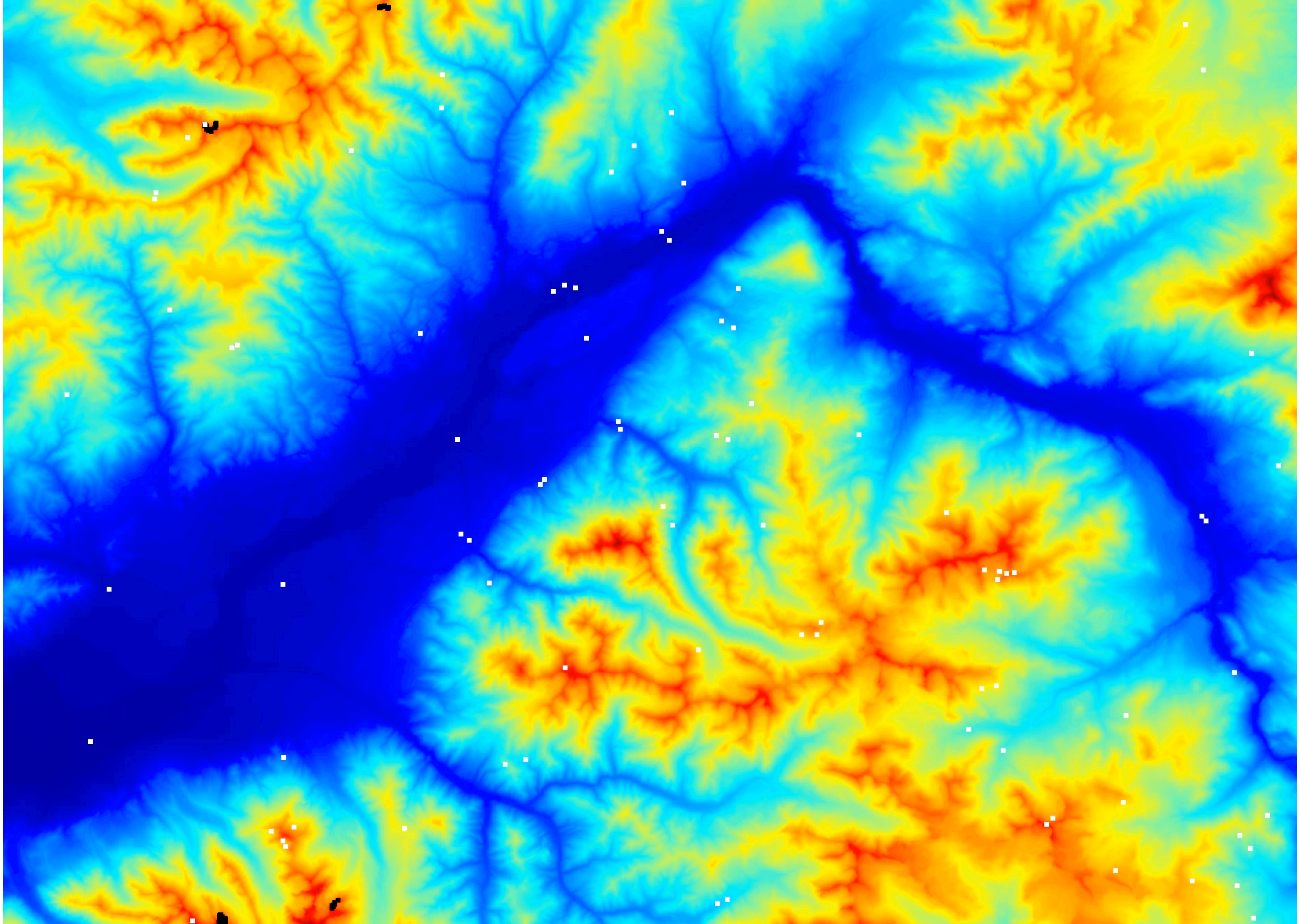
[Cornish & Porter arXiv:gr-qc/0612091 , Fairhurst et al arXiv:1908.05707]

Parallel Tempering

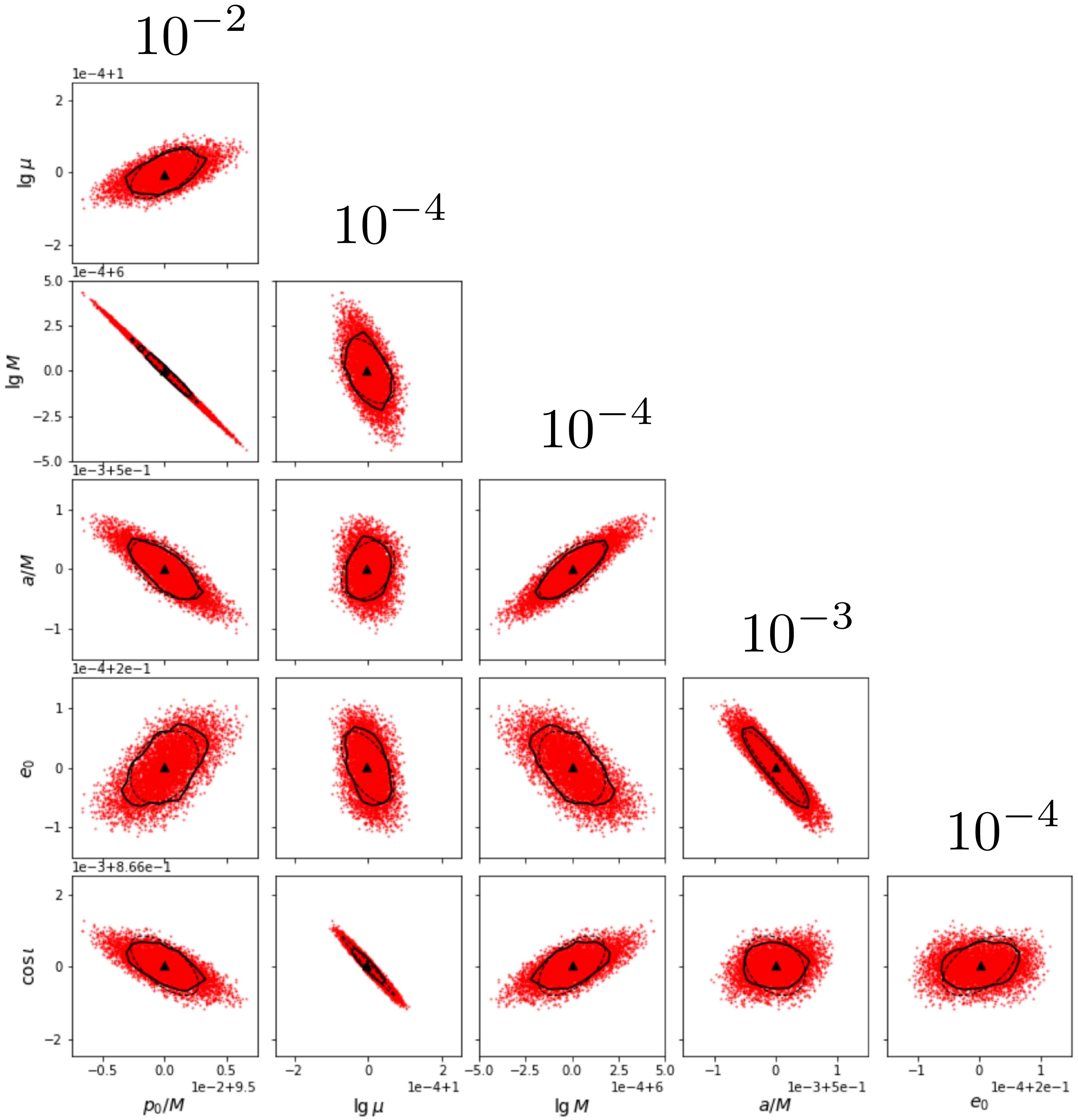




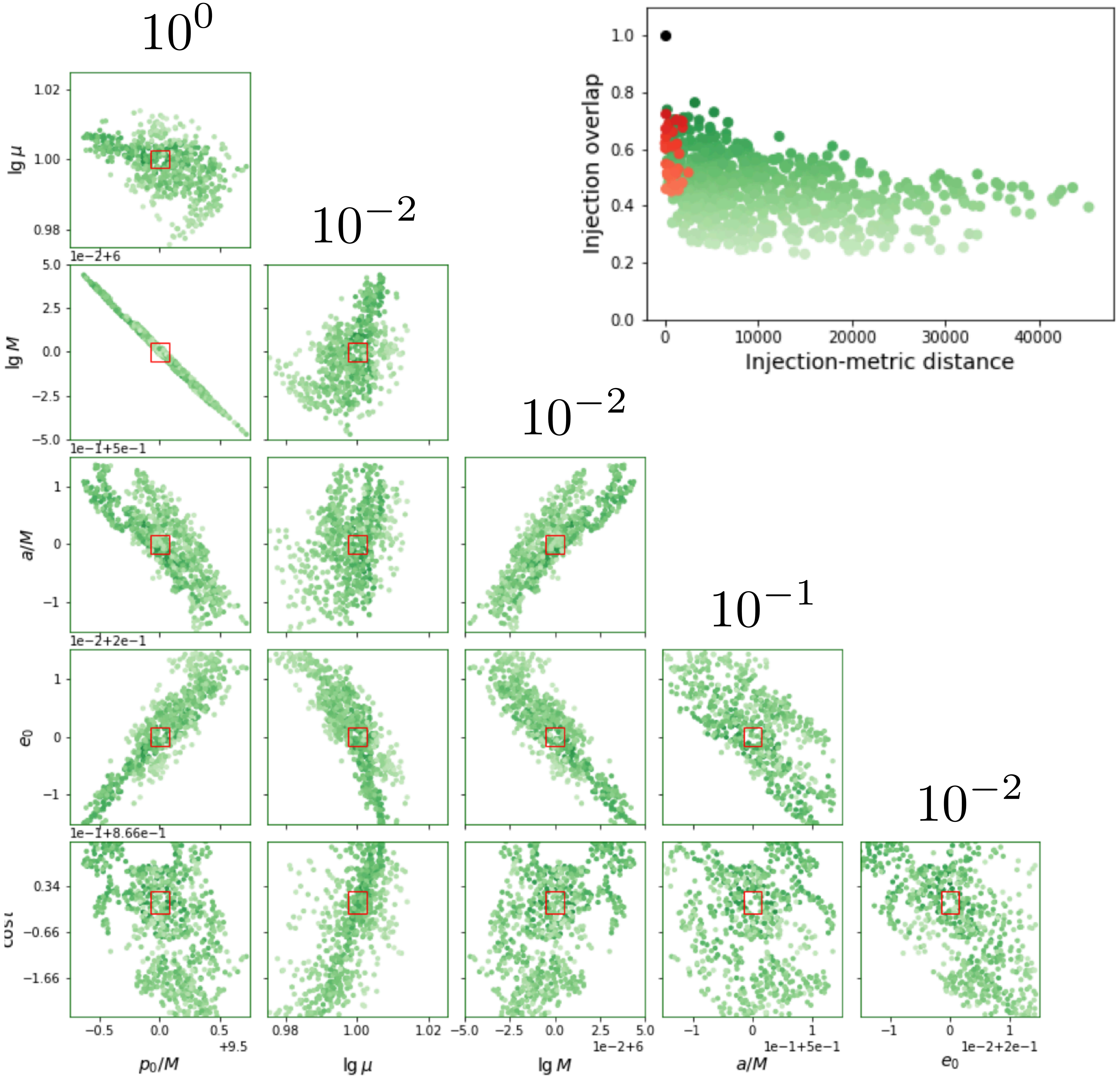
$$\mathrm{SNR}^2_{\mathrm{eff}} = \frac{\mathrm{SNR}^2}{T}$$



Secondary maxima - Blessing or Curse?



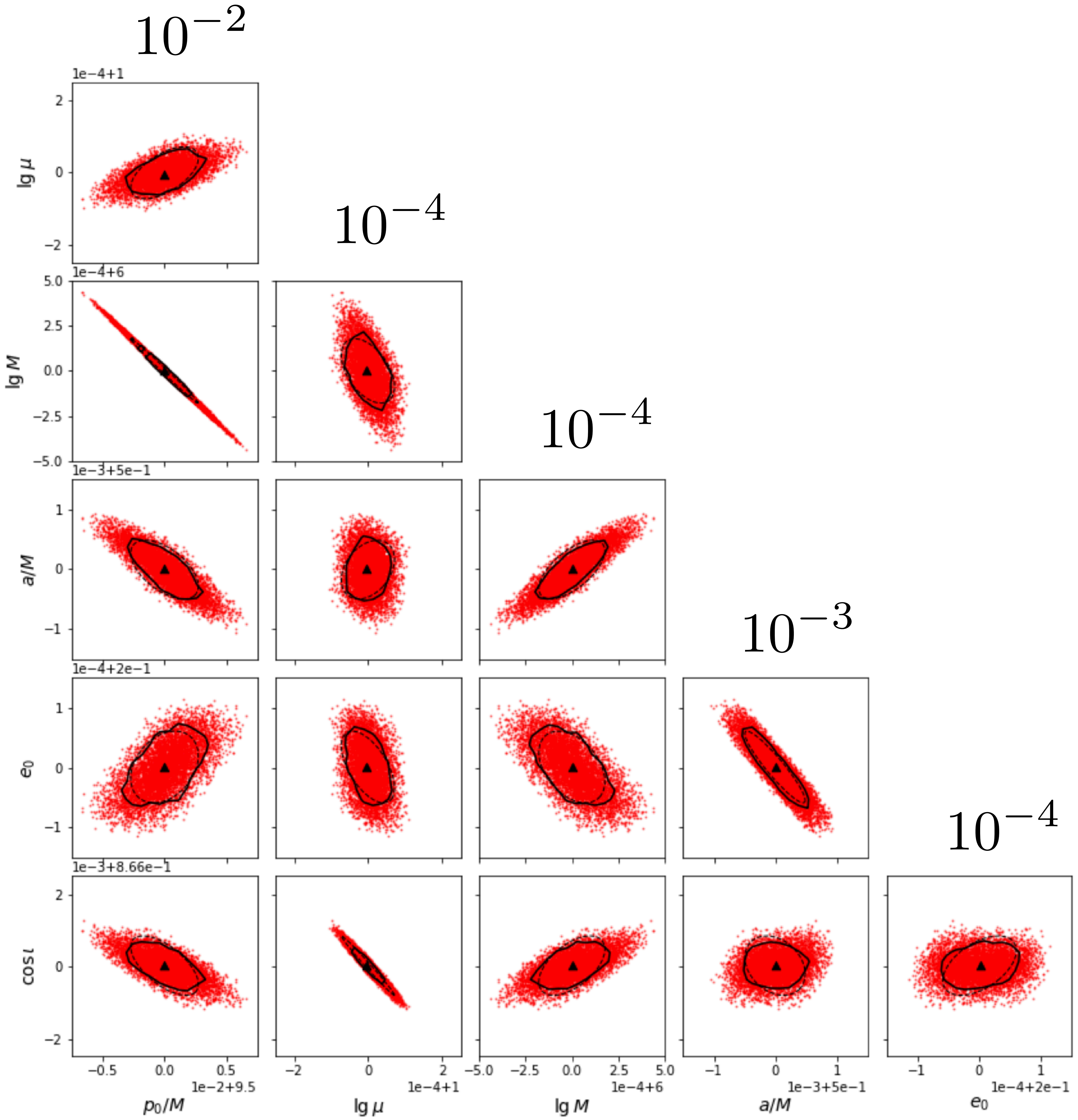
Posterior

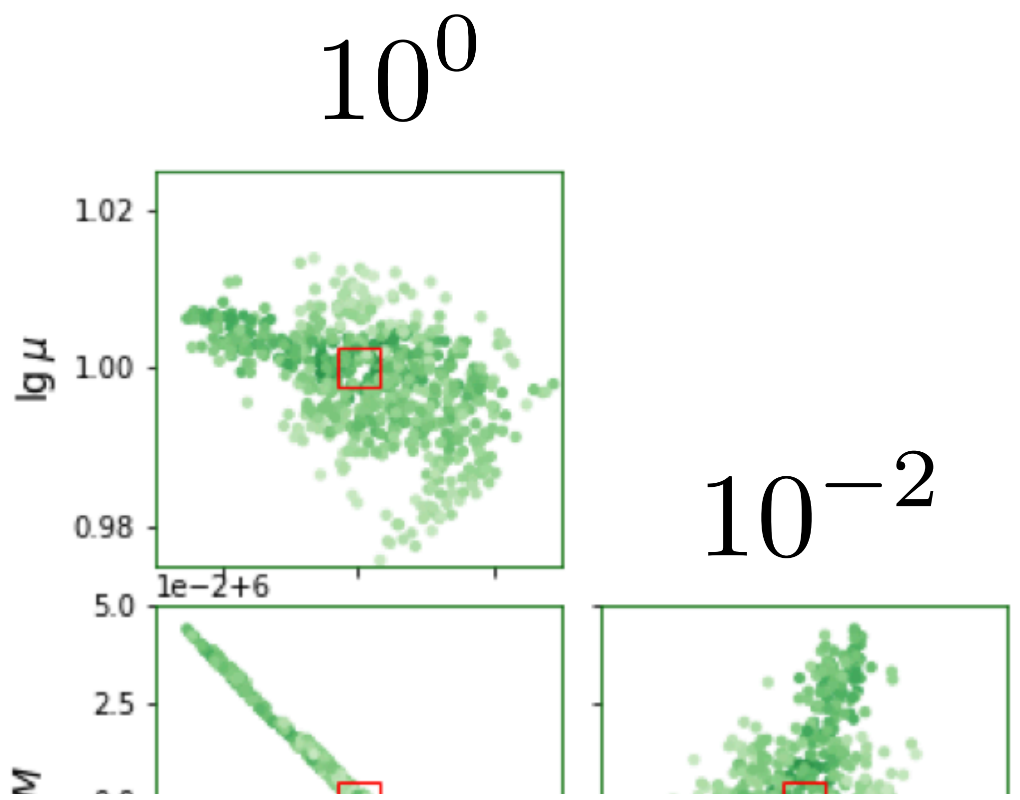


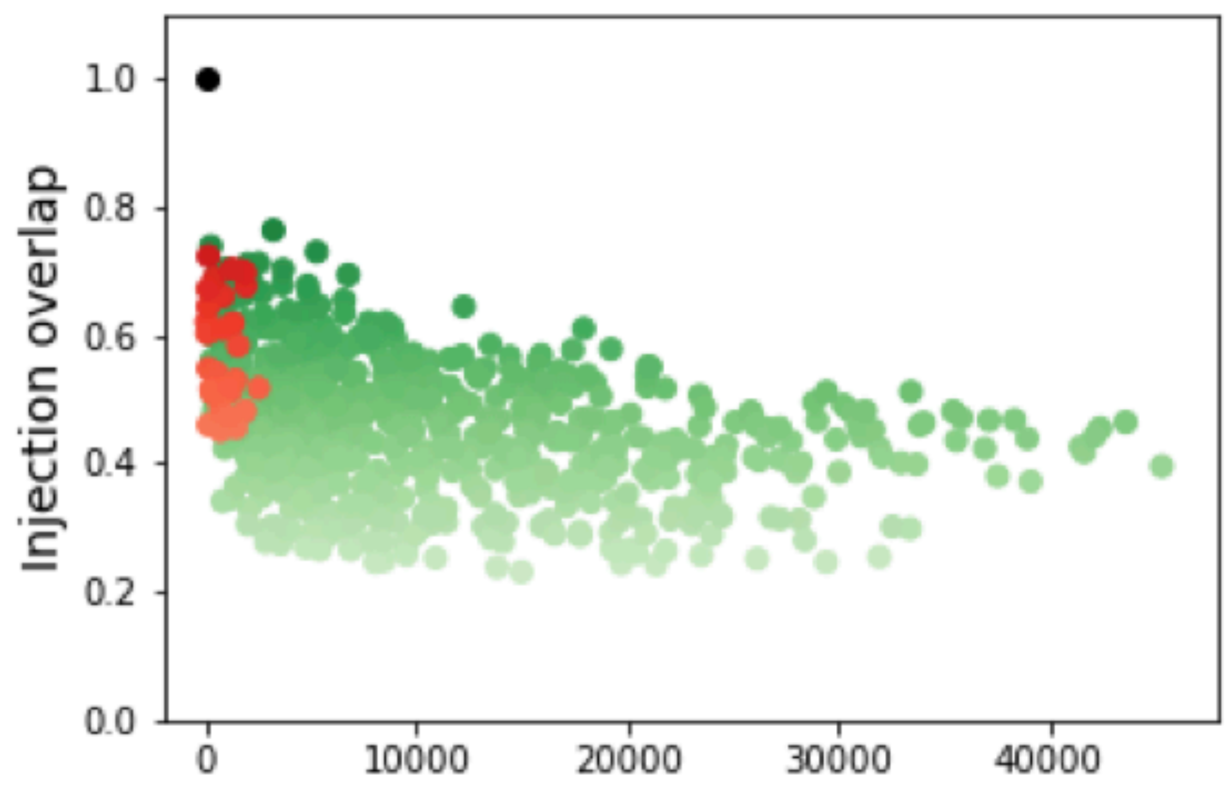
[Chua & Cutler 2109.14254]

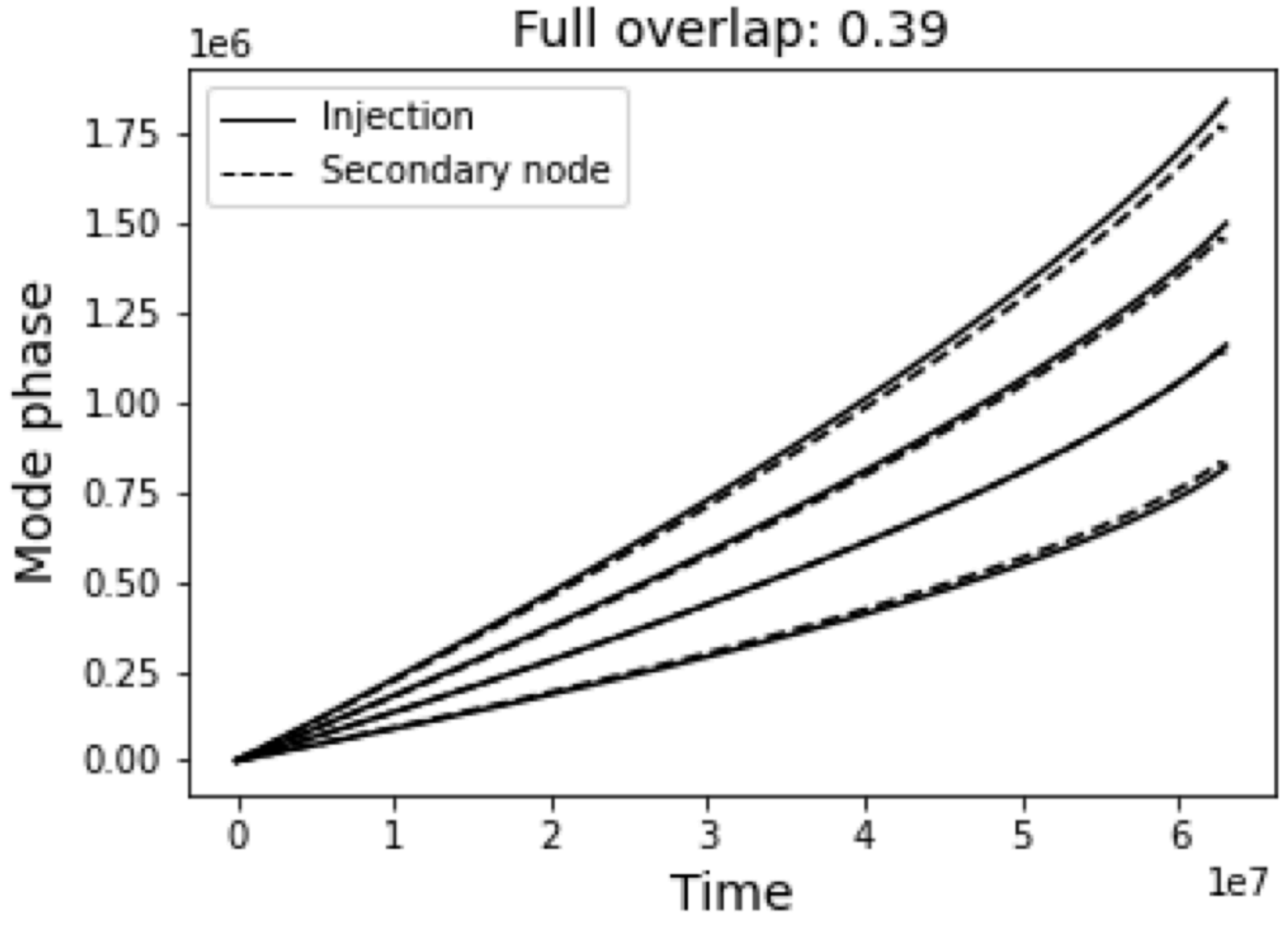
Secondaries

Secondary maxima - Blessing or Curse?









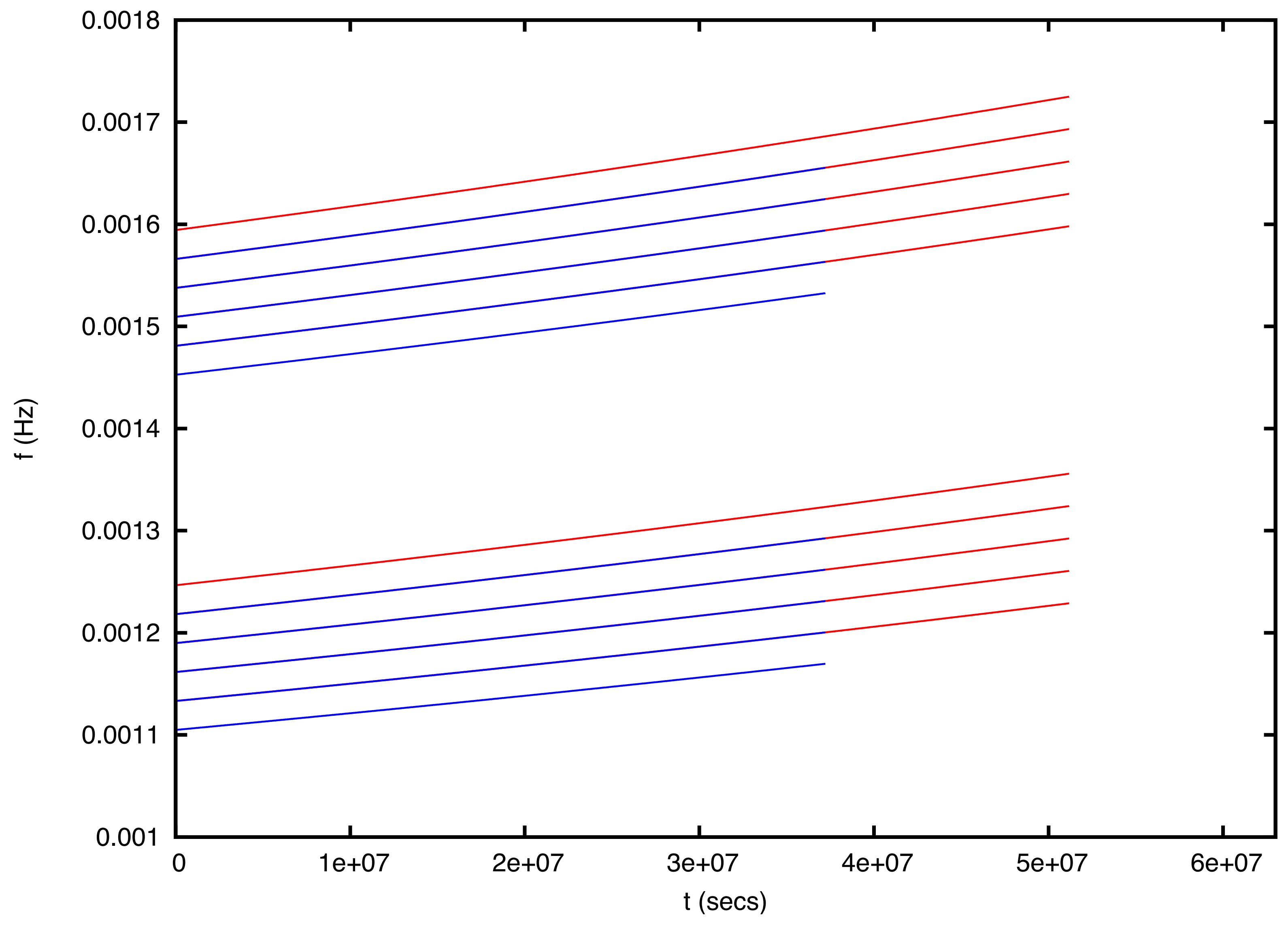
Posterior

[Chua & Cutler 2109.14254]

Secondaries

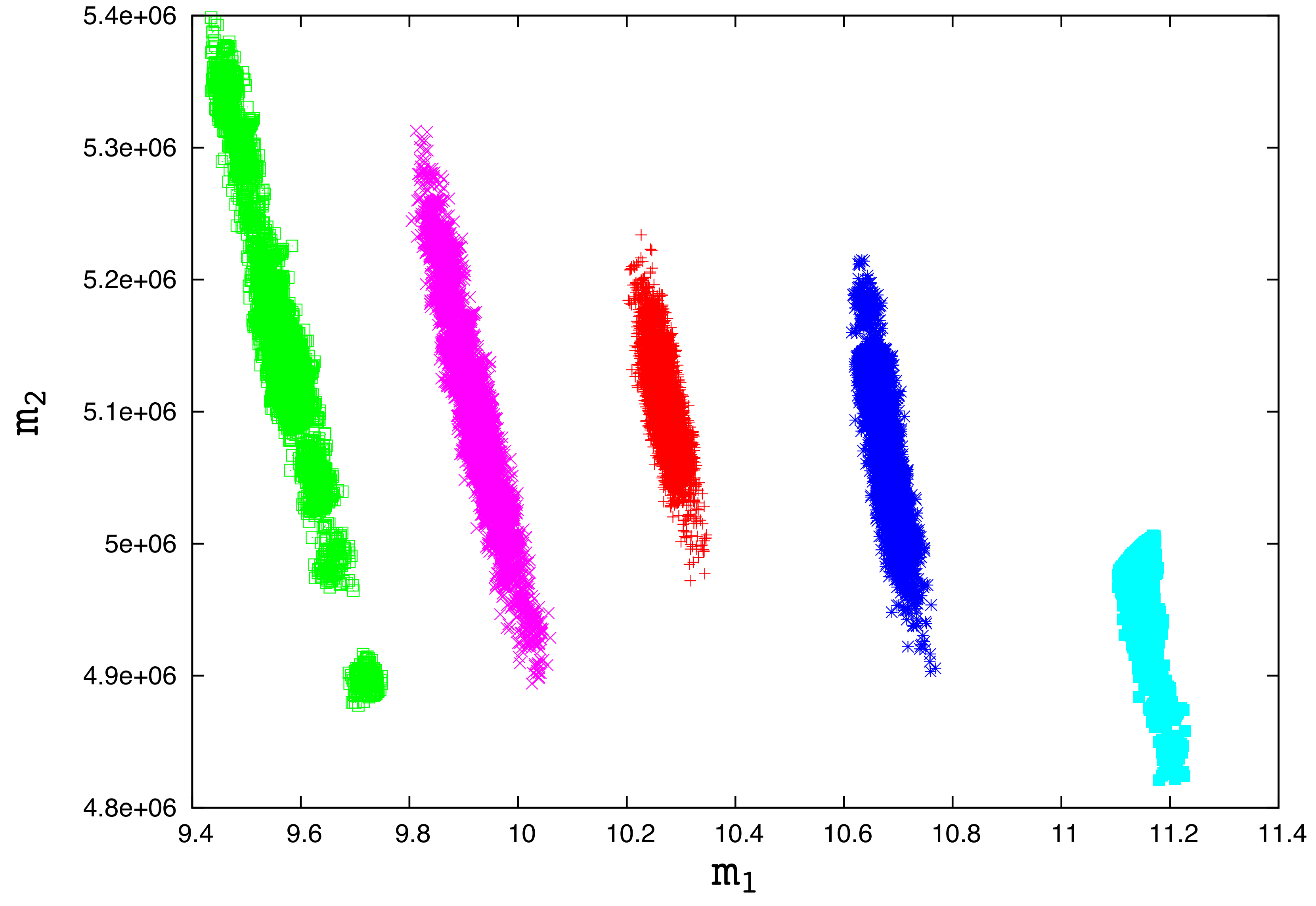
Secondary maxima can be a good thing

[Cornish, 0804.3323]

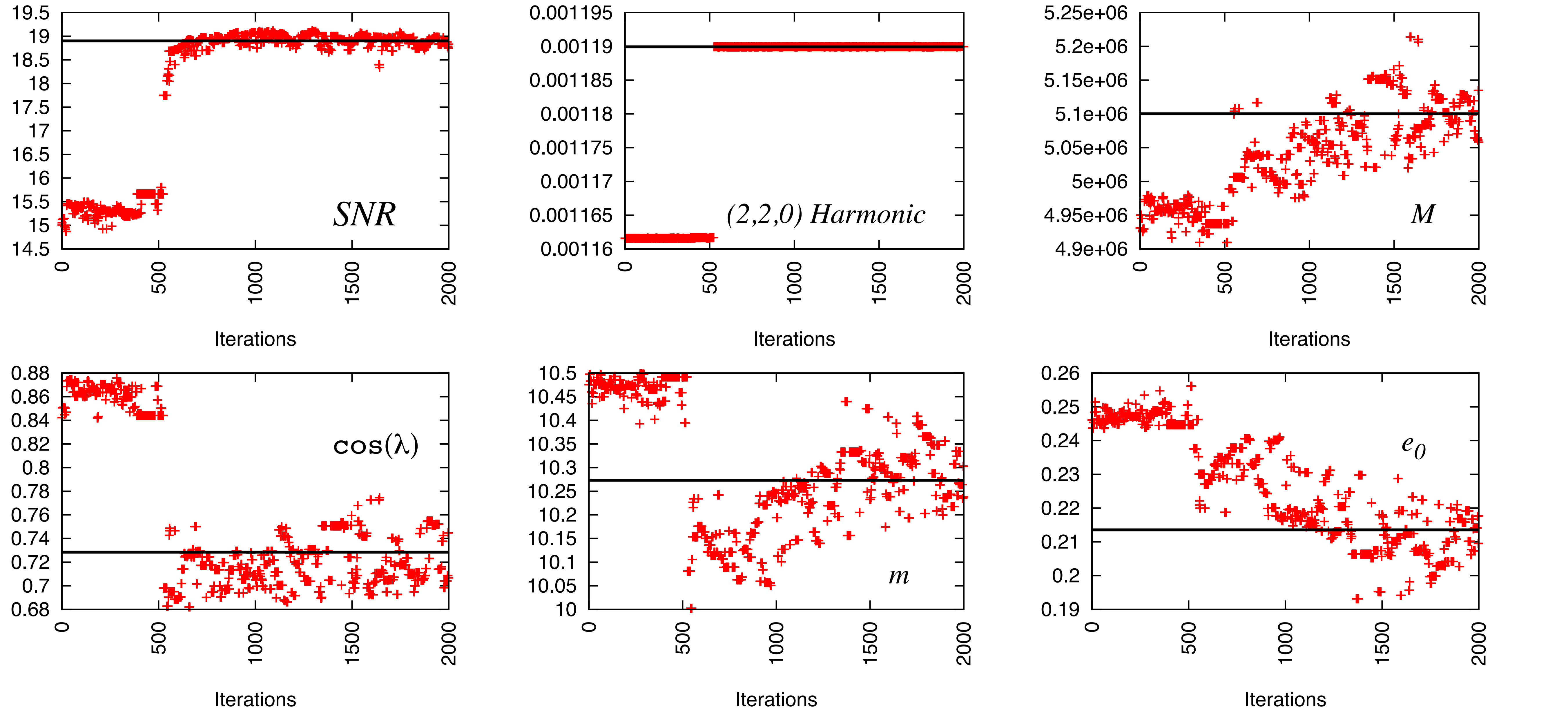


MCMC within an MCMC - Use a fast track matching likelihood to jump from one solution to the other (the "data" is the current solution)

$$\chi^2 = \sum_{nmk} \left[(f_{nmk} - f'_{nmk})^2 T^4 + (\dot{f}_{nmk} - \dot{f}'_{nmk})^2 T^6 + \frac{1}{4} (\ddot{f}_{nmk} - \ddot{f}'_{nmk})^2 T^8 \right]$$

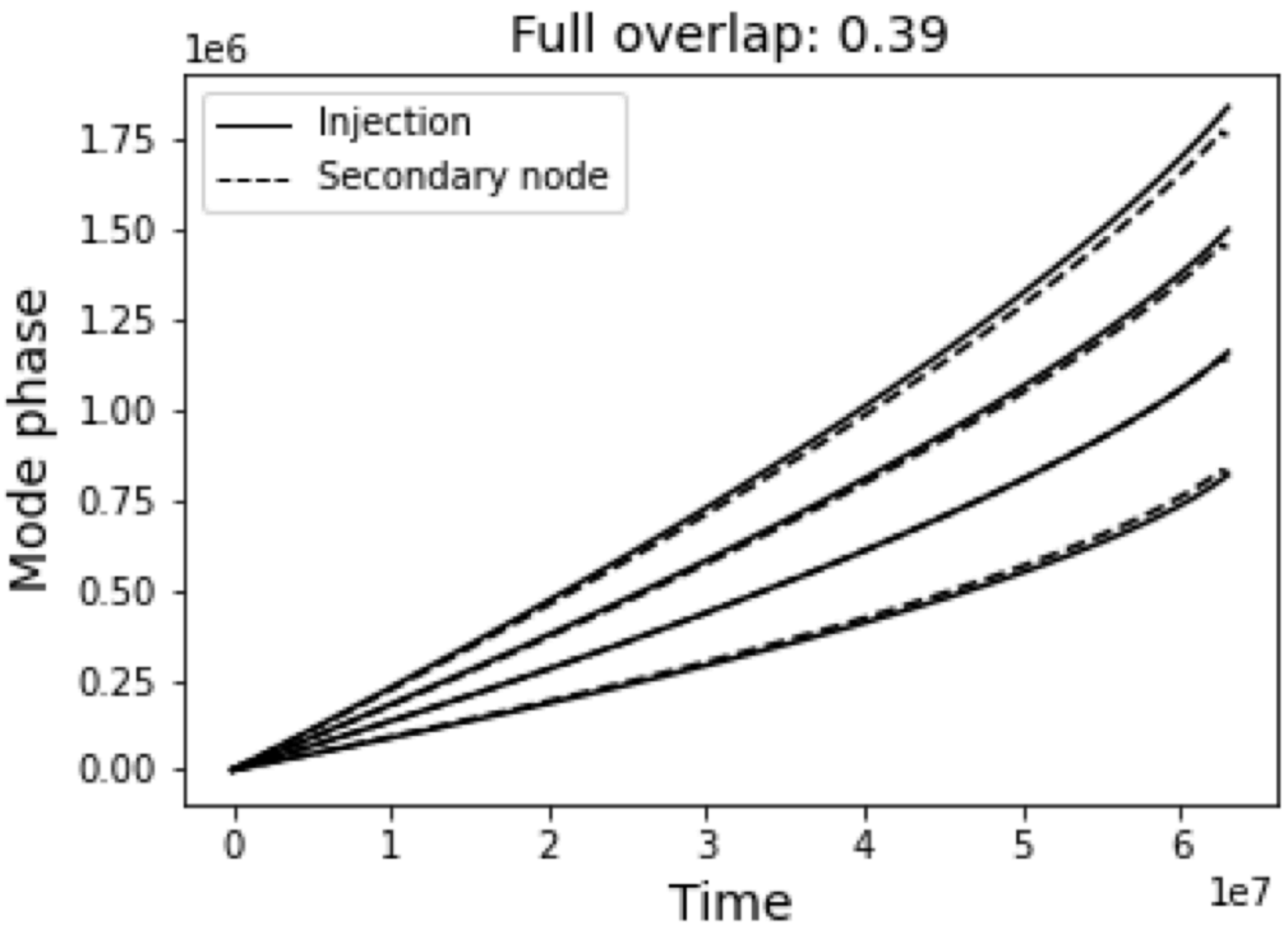


Use the posteriors from this track matching MCMC as a proposal in the full MCMC



The Secondary of a Secondary can be the Primary

Chua & Cutler secondaries



Suppose the search is stuck on a secondary maxima

Use this template as data and do an MCMC within the MCMC to find its secondaries

Nearby in phase so can use *very* fast noise-free heterodyned likelihood [Cornish 1007.4820]

Use the posterior from this fast MCMC as a proposal for the main MCMC. One of the secondaries of the current solution will be the primary

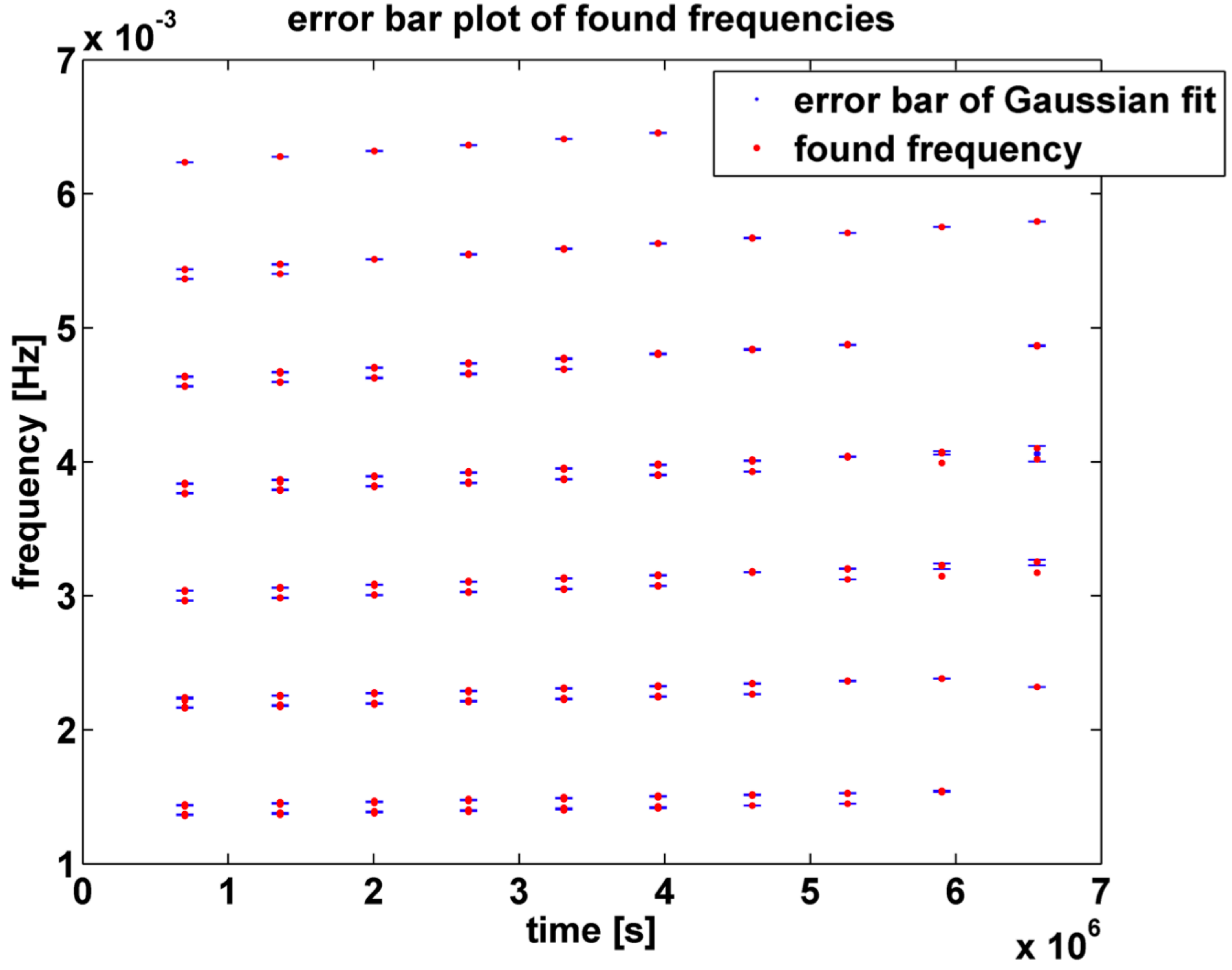
Phenomenological Models for Detection

$$\Phi_r(t) = \Phi_r(t_0) + \omega_r(t_0)(t-t_0) + \frac{1}{2}\dot{\omega}_r(t-t_0)^2 + \ldots$$
$$= \Phi_r(t_0) + 2\pi f_r(t_0)(t-t_0) + \pi \dot{f}_r(t-t_0)^2 + \ldots$$

$$\Phi_\theta(t) = \Phi_\theta(t_0) + \omega_\theta(t_0)(t-t_0) + \frac{1}{2}\dot{\omega}_\theta(t-t_0)^2 + \ldots$$
$$= \Phi_\theta(t_0) + 2\pi f_\theta(t_0)(t-t_0) + \pi \dot{f}_\theta(t-t_0)^2 + \ldots$$

$$\Phi_\varphi(t) = \Phi_\varphi(t_0) + \omega_\varphi(t_0)(t-t_0) + \frac{1}{2}\dot{\omega}_\varphi(t-t_0)^2 + \ldots$$
$$= \Phi_\varphi(t_0) + 2\pi f_\varphi(t_0)(t-t_0) + \pi \dot{f}_\varphi(t-t_0)^2 + \ldots$$

$$A_{lmn}(t) = A_{lmn}(t_0) + \dot{A}_{lmn}(t_0)(t-t_0) + \ldots$$

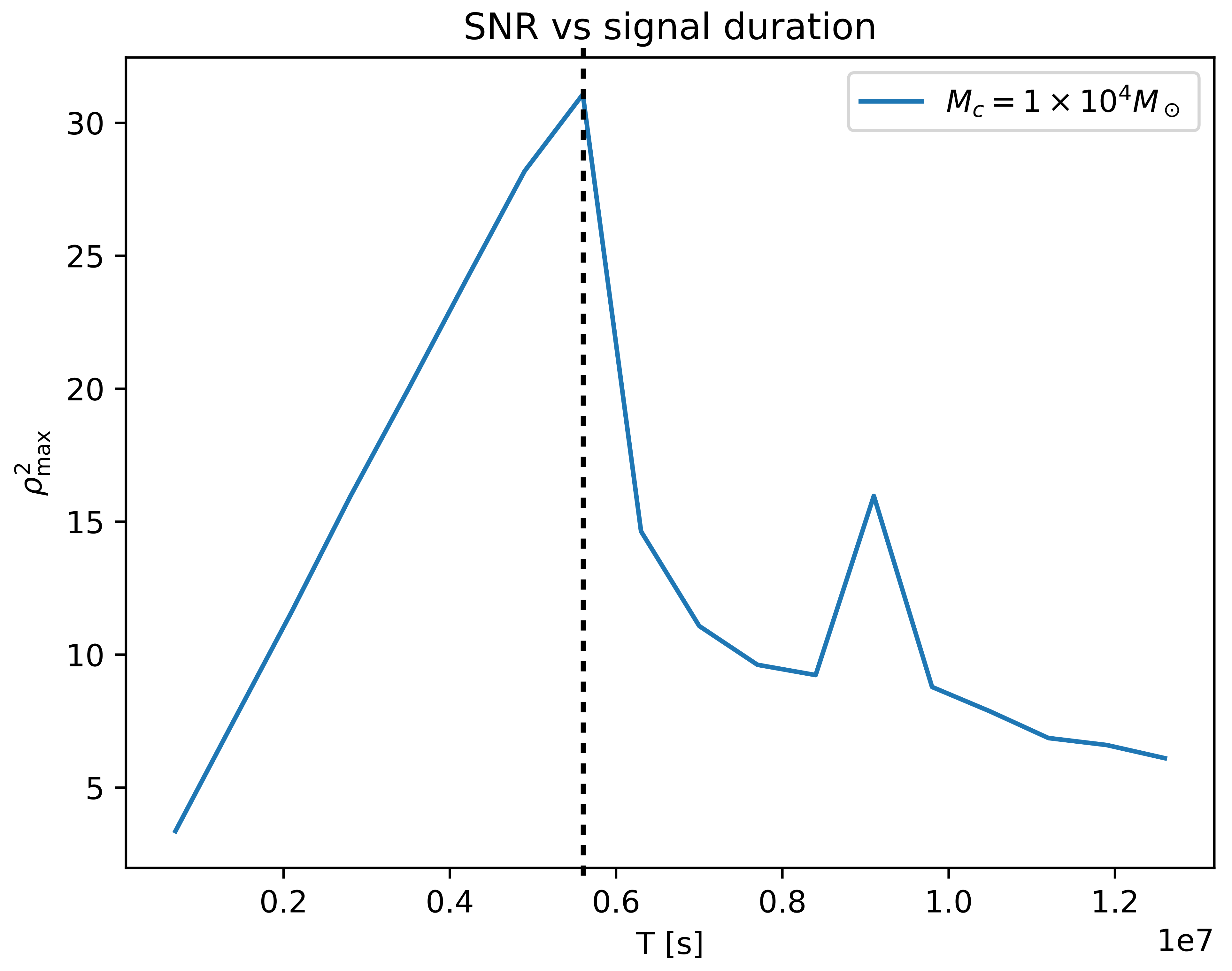


Challenge is mapping phenomenological parameters to physical parameters

[Wang, Shang & Babak, arXiv 1207.495]

Phenomenological search (IMBHB example)

[Dey & Cornish in progress]



Can ignore LISA orbital modulation for integrations less than a few weeks

Example: fitting a PN inspiral with a linear chirp

Optimal coherent integration duration

Read the (old) literature!

"Those who cannot remember the past are condemned to repeat it."

George Santayana

LISA EMRI Hunt Papers

"Event rate estimates for LISA extreme mass ratio capture sources", Gair et al, arXiv 0405137

"Detection Strategies for Extreme Mass Ratio Inspirals", Cornish, arXiv 0804.3323

"An Algorithm for detection of extreme mass ratio inspirals in LISA data", Babak, Gair & Porter, arXiv 0902.4133

"EMRI data analysis with a phenomenological waveform", Wang, Shang & Babak, arXiv 1207.495

LIGO Continuous Wave Papers

"Searching for periodic sources with LIGO", Brady et al, arXiv 9702050

"Searching for continuous gravitational wave signals: The hierarchical Hough transform algorithm", Papa & Schutz, arXiv 0011034

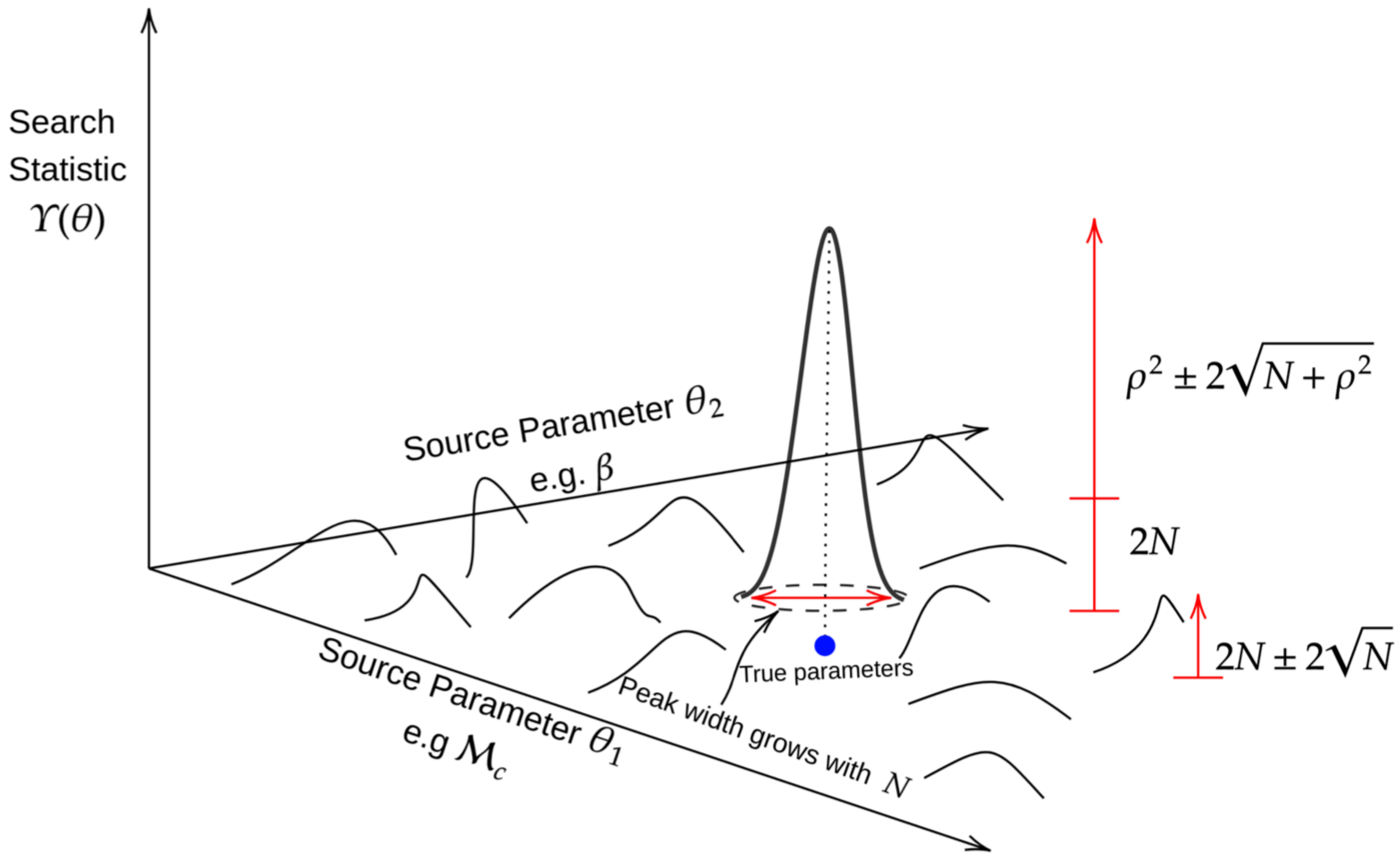
"Data analysis of gravitational-wave signals from spinning neutron stars…", Jaranowski, et al, arXiv 9804014

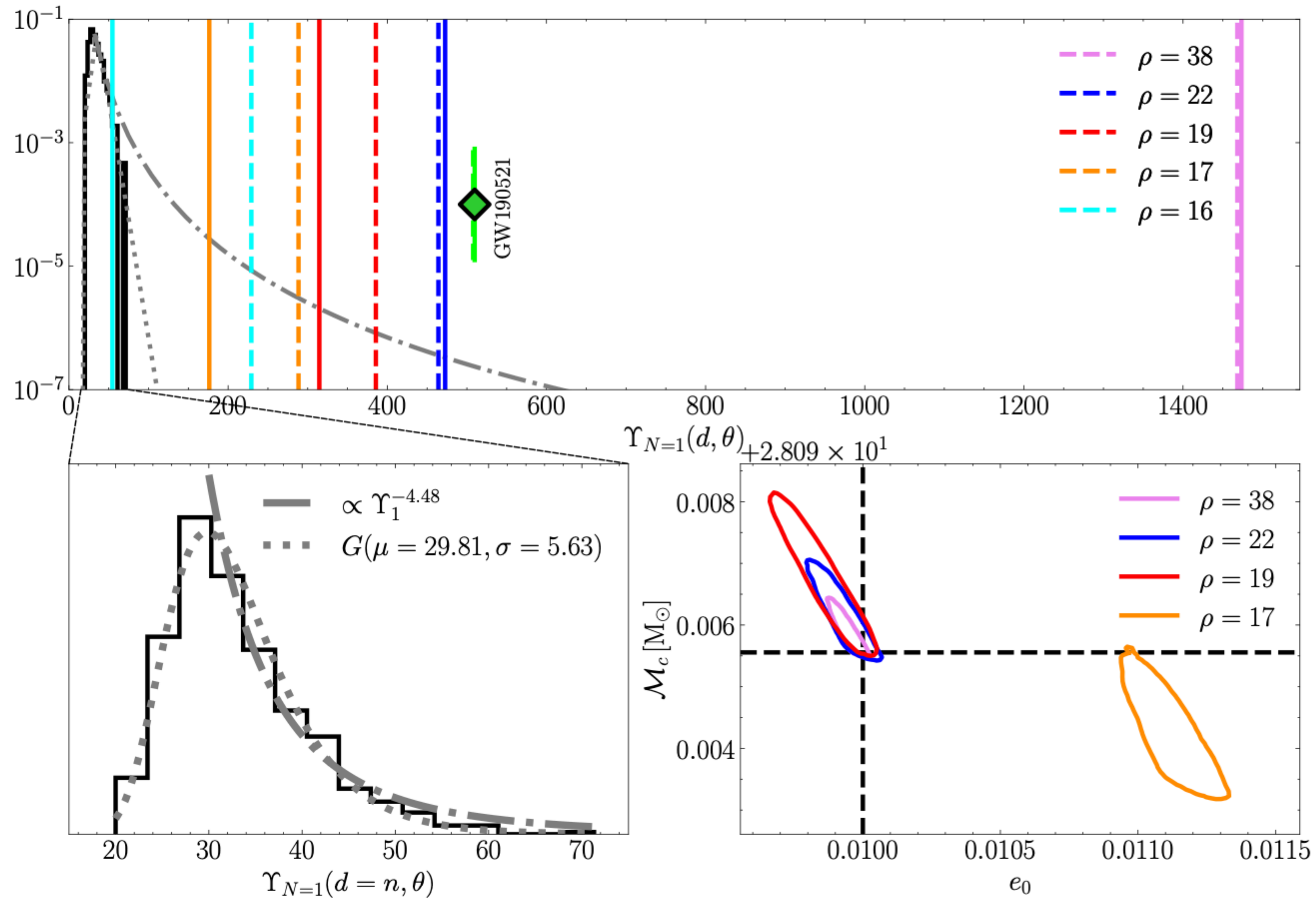
"On blind searches for noise dominated signals: a loosely coherent approach", Dergachev, arXiv 1003.2178

Recent Work

Stellar Origin Black Holes in LISA - “baby EMRIs”

[Bandopadhyay & Moore 2412.10501]

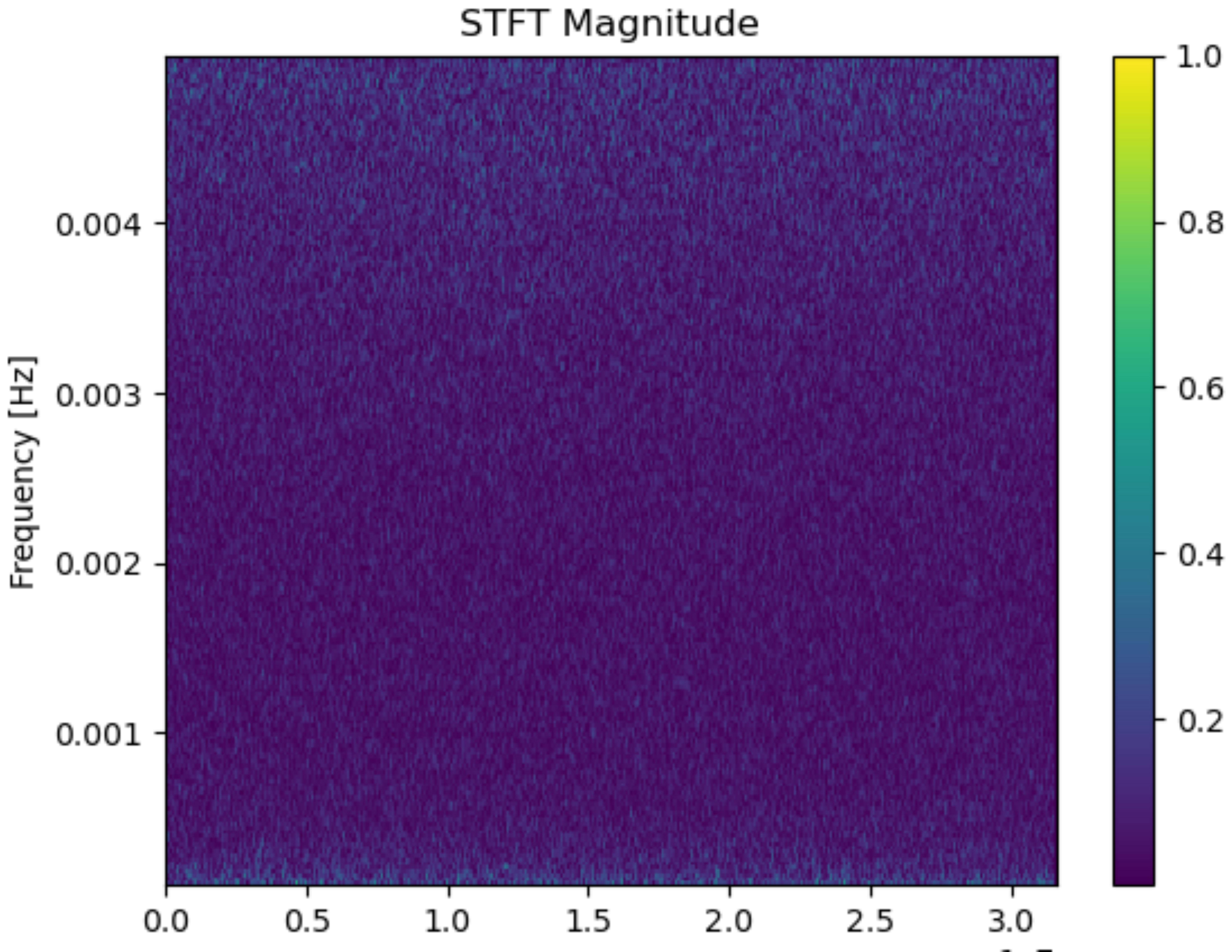




Semi-coherent. Amplitude and phase maximized likelihood in each segment.
Particle swarm search. GPU accelerated

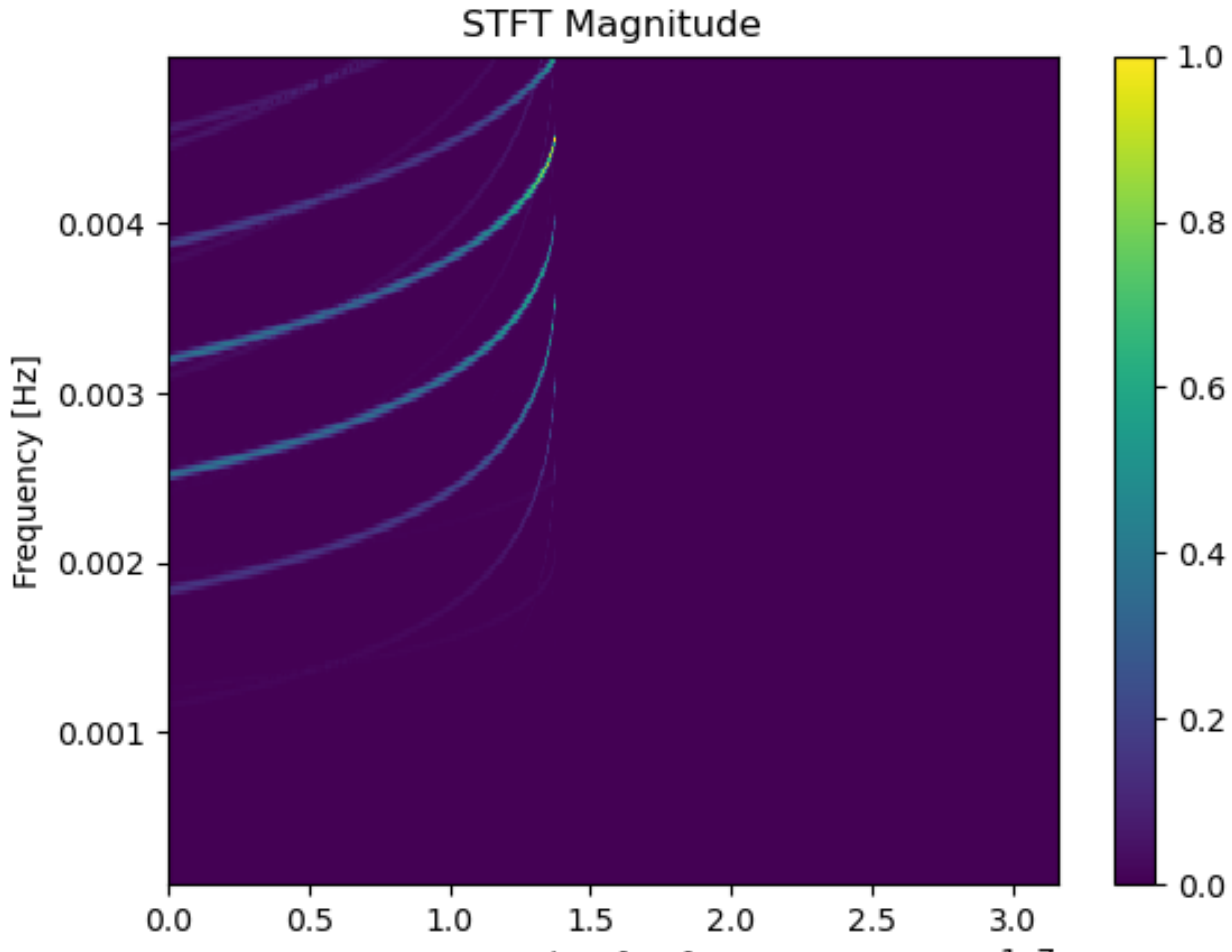
Semi-coherent search with the Fast EMRI Waveform model

[Lorenzo Speri in progress]



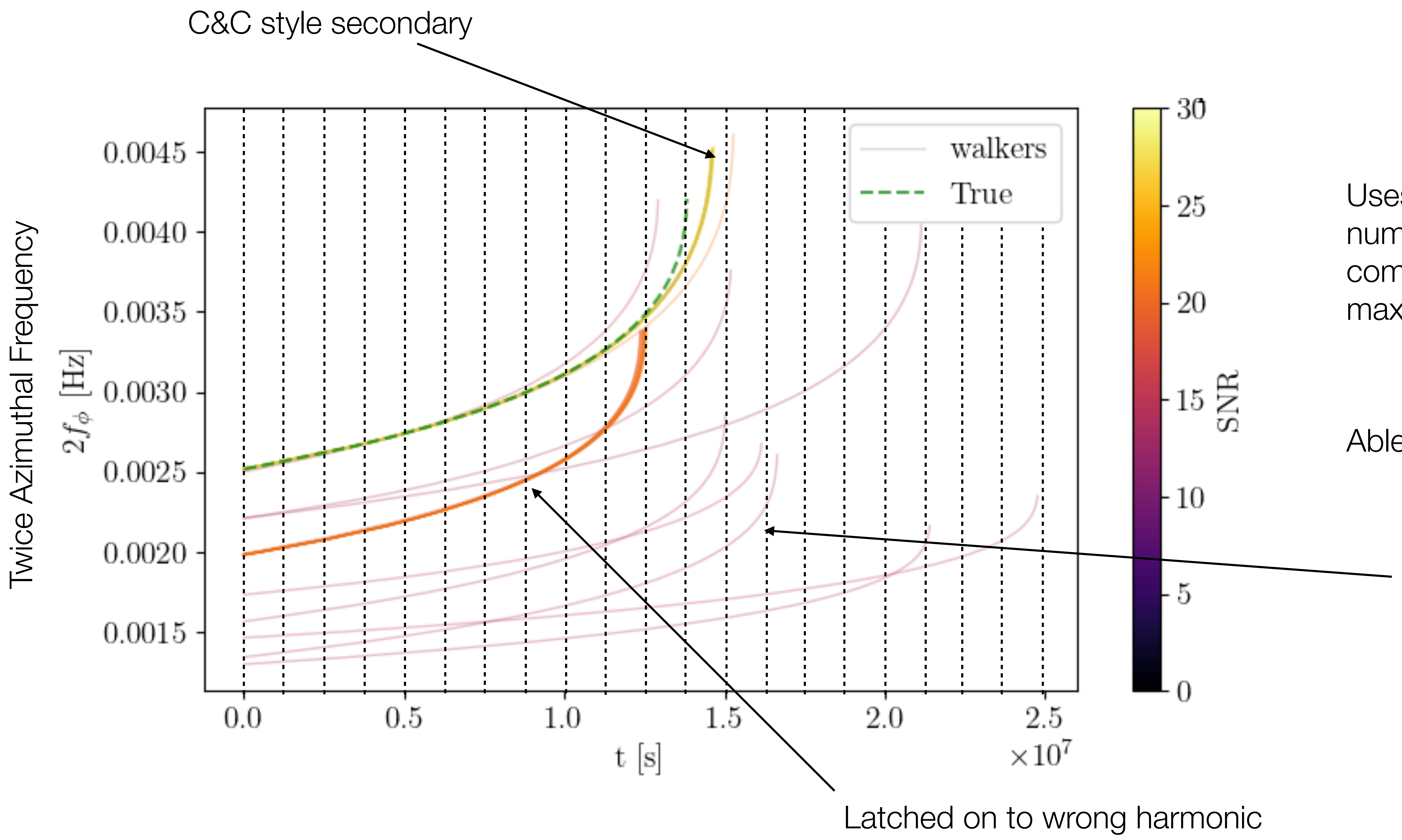
Semi-coherent search with the Fast EMRI Waveform model

[Lorenzo Speri in progress]



Semi-coherent search with the Fast EMRI Waveform model

[Lorenzo Speri in progress]



Uses full waveform model (but reduced number of harmonics). Likelihood computed using 2-week chunks, maximized wrt overall amplitude and phase

Able to find high SNR~100 EMRIs

Backup Slides